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SPHysics-FUNWAVE hybrid model for coastal wave propagation

Modèle hybride SPHysics-FUNWAVE pour la propagation côtière des vagues

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ABSTRACT

It is difficult to study the process of wave propagation from the deep ocean to the nearshore region using a single model due to the presence of multiple scales both in time and in space. Numerical models based on the Boussinesq equations are well known to accurately propagate waves from intermediate water depth to the nearshore region. Since they are 2D models, they are computationally efficient and can be applied to study wave transformations over large domains. Numerical models based on Smoothed Particle Hydrodynamics can inherently capture multiply connected free surfaces and hence can be naturally used to capture breaking free surfaces and estimate breaking induced runup and overtopping. Here, a hybrid model (SPHunwave) is developed combining the main advantages of a Boussinesq model (FUNWAVE) and a SPH model (SPHysics). The details of the coupling procedure along with preliminary validation tests are presented.

RÉSUMÉ

Il est difficile d'étudier le processus de propagation des vagues depuis l'océan profond jusqu'au bord du rivage avec un seul modèle en raison de la présence de multiples échelles de temps et d'espace. Les modèles numériques basés sur les équations de Boussinesq sont bien connus pour propager correctement les vagues depuis une profondeur intermédiaire de l'eau jusqu'à la région proche du rivage. Comme ce sont des modèles 2D, ils sont informatiquement efficaces et peuvent être appliqués à l'étude de l'évolution des vagues sur de grands domaines. Les modèles numériques basés sur la formulation SPH peuvent intrinsèquement gérer des surfaces libres multiplement connexes, ils peuvent donc naturellement traiter les déferlements et permettre l'évaluation du runup et du déversement induits. On développe ici un modèle hybride (SPHunwave) qui combine les principaux avantages d'un modèle de Boussinesq (FUNWAVE) et d'un modèle SPH (SPHysics). Les détails du procédé de couplage sont présentés ainsi que les premiers tests de validation.

Keywords: Boussinesq type wave propagation model, coupling technique, FUNWAVE, hybrid model, smoothed particle hydrodynamics, SPH, SPHysics

1 Introduction

A variety of oceanographic models using different numerical and physical approaches have been developed to handle wave propagation and different types of wave transformations as refraction, diffraction, breaking, run-up and overtopping. An overview of different models to describe wave behavior can be seen in Liu and Losada (2002). These models have several advantages and limitations that depend on their mathematical formulations and numerical implementations.

Recently, research has been focused on coupling models that employ different numerical and mathematical approaches (Cui and Williams 1998, Pun *et al.* 2000, Soares-Frazão and Zech 2002). The primary goal of such an approach is to combine the advantages of the individual models in a single model, thus increasing the accuracy, efficiency and regime of validity. Flekkoy *et al.* (2000) combined a compressible Navier Stokes model to a molecular dynamics model to study both macroscopic and microscopic behaviors of a fluid. They achieved the coupling by using two planes in the computational domain to exchange

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information between the models about mass and momentum flux. A similar hybrid model was developed by Nie *et al.* (2004) wherein they used an overlap region to exchange information between the models. They used a constrained dynamics approach that ensured mass and momentum conservation. In coastal engineering, Sitanggang *et al.* (2006) have coupled a Boussinesq model and a Reynolds Averaged Navier Stokes (RANS) model. They have used this model successfully to simulate solitary wave propagation and standing waves.

Depth averaged models such as models based on the nonlinear shallow water equations, and Boussinesq equations are widely used to describe wave transformation from the offshore to the nearshore region. They have been successfully used to study tsunami propagation, wave induced nearshore currents and run-up. These models are attractive because they can accurately describe wave propagation, and are computationally efficient when compared to fully three dimensional models. This makes it feasible to use such models to study wave transformation over large domains. However, using these models it is difficult to study the details of 3D flow features such as those observed during the wave breaking process.

A new approach that combines the advantages of two existing wave propagation models is proposed. This hybrid model couples the finite difference Boussinesq model FUNWAVE (Kirby *et al.* 1998) to SPHysics (Gómez-Gesteira *et al.* 2009), a Smoothed Particle Hydrodynamics (SPH) model. The aim of this paper is the development of a hybrid model that employs FUNWAVE for wave propagation in the offshore region and uses SPHysics to handle wave transformation in the nearshore. The methodology of coupling the two models is presented and the results from a simulation of solitary wave propagation in a tank with constant depth are discussed.

2 Coupling model

2.1 *Boussinesq models: FUNWAVE*

The standard form of the Boussinesq equations were derived by Peregrine (1967) for variable depth assuming that both frequency dispersion and nonlinearity were weak and comparable. Alternative forms of Boussinesq equations have been derived by Madsen *et al.* (1991), Nwogu (1993) and Wei *et al.* (1995) to extend the validity of the method and include new physics. Numerical results based on Boussinesq approach, both on its standard or extended formulations have shown a high accuracy when compared with experimental data (Stansby 2003).

There are two dimensionless parameters that are important to describe the characteristics of long wave models. These are $\varepsilon = ah^*$ and $\mu^* = kh$, where a is the incident wave amplitude, h^* is the reference water depth and k is the reference wave number. Wei and Kirby (1995) derived a set of Boussinesq equations to include all nonlinear terms and dispersive terms to order μ^{*2} and $\varepsilon\mu^{*2}$. This ensured that these equations were fully nonlinear with respect to the assumptions made i.e. all the terms, arising out of the expansion of the linear dispersion relationship, describing the nonlinear effects were included. A finite difference model of these fully nonlinear Boussinesq equations called FUNWAVE was developed by them and is used as the Boussinesq model in this paper. A detailed description and validation tests of the model can be found in the FUNWAVE manual (Kirby *et al.* 1998).

The fully nonlinear Boussinesq equations solve the surface elevation η and the velocity field $u_\alpha = u(x, z_\alpha)$ evaluated at a arbitrary elevation $z = z_\alpha(x)$ and are given by

$$\eta_t + \nabla \cdot M = 0 \tag{1}$$

$$u_{\alpha t} + (u_\alpha \cdot \nabla)u_\alpha + g\nabla\eta + V_1 + V_2 = 0 \tag{2}$$

where $u\alpha = u(x, z\alpha)$ and the subscript 't' represent the partial time derivative.

A right handed coordinate system is chosen such that x is the horizontal coordinate and the positive z-axis is directed upwards with $z = 0$ on the still water surface.

The terms M, V_1, V_2 are given by

$$M = M_1 + M_2 \tag{3}$$

$$M_1 = (h^* + \eta)u_\alpha \tag{4}$$

$$M_2 = (h^* + \eta)\left\{\left[\frac{1}{2}z_\alpha^2 - \frac{1}{6}(h^{*2} - h^*\eta + \eta^2)\right]\nabla(\nabla u_\alpha) + \left[z_\alpha + \frac{1}{2}(h^* - \eta)\right]\nabla\left[\nabla(h^* u_\alpha)\right]\right\} \tag{5}$$

$$V_1 = \frac{1}{2}z_\alpha^2\nabla(\nabla u_{\alpha t}) + z_\alpha\nabla\left[\nabla(h^* u_{\alpha t})\right] - \nabla\left[\frac{1}{2}\eta^2\nabla u_{\alpha t} + \eta\nabla(h^* u_{\alpha t})\right] \tag{6}$$

$$V_2 = \nabla\left\{(z_\alpha - \eta)(u_\alpha \cdot \nabla)\left[\nabla(h^* u_\alpha)\right] + \frac{1}{2}(z_\alpha^2 - \eta^2)(u_\alpha \cdot \nabla)(\nabla u_\alpha)\right\} + \frac{1}{2}\nabla\left\{\left[\nabla(h^* u_{\alpha t}) + \eta\nabla \cdot u_\alpha\right]^2\right\} \tag{7}$$

M_2, V_1, V_2 are the additional terms that account for the nonlinear and dispersive effects and are obtained as a result of the approximation of the form of the velocity potential. It is clear that neglecting M_2, V_1, V_2 would reduce the system to a set of nonlinear shallow water equations. In these equations, the value of the free parameter z_α needs to be specified. By minimizing the errors between the linear dispersion relationship obtained from the standard Boussinesq equations and the linear dispersion relationship of Stokes waves, Nwogu (1993) suggested that $z_\alpha = -0.531h(x)$ be used to obtain a best fit for the phase velocities.

2.2 *SPH models: SPHysics code*

SPHysics is a Smoothed Particle Hydrodynamics (SPH) model of the Navier-Stokes equations (Monaghan 1992) developed to study free-surface flows. It is the product of a collaborative effort amongst researchers at the Johns Hopkins University (US), the

University of Vigo (Spain) and the University of Manchester (UK). The first version of SPHysics was released on August 2007 and is available for public use at www.sphysics.org. A complete description of the software is available in the SPHysics user's guide (Gómez-Gesteira *et al.* 2009).

SPHysics can simulate various commonly observed free-surface flow phenomena such as dam breaks, landslides, sloshing in tanks and wave impacts on structures. The model, written in FORTRAN90, is in a modular form enabling users to add and remove features as desired. Consequently, the user has the flexibility to choose from a variety of kernel functions, kernel corrections, density filters, viscosity treatments, boundary conditions, time schemes, operating systems and compilers. The options chosen to run the simulations presented in this paper are briefly described below.

Capone *et al.* (2007) performed a set of numerical tests to assess the performance of different existing smoothing kernels commonly used in SPH. They concluded that the quintic Wendland kernel (Wendland 2005) was accurate, easy to implement and computationally inexpensive. Hence, the Wendland kernel is used in the simulations presented herein. However the Wendland kernel, as with a majority of kernels in SPH, suffers from tensile instability (Swegle *et al.* 1995) which could potentially result in particle clumping or explosion during a numerical simulation. The empirical tensile instability correction proposed by Monaghan (2000) is used to treat this problem. It is well known that the standard SPH interpolation introduces errors at the boundaries where the compact support of the kernel is incomplete. The mixed kernel and gradient corrections formulated by Chen and Beraun (2000) to ensure accurate estimation of functions and their gradients are used to ensure accurate interpolations near the boundaries. Repulsive forces (Monaghan and Kos 1999) are implemented to describe the boundary conditions. In order to adequately represent the laminar viscous effects, the laminar stress terms are discretized according to Morris *et al.* (1997). Viscous dissipation due to turbulent motions in the fluid are included in SPHysics by Favre-averaging the governing equations Dalrymple and Rogers (2006) and Shao and Gotoh (2005). The turbulent stresses in this Favre-averaged sub particle scale model are modeled using a Smagorinsky (Smagorinsky 1963) eddy viscosity technique. In order to maintain a stable solution, the density of the particles are reinitialized using a Shepard filter (Panizzo 2004, Colagrossi and Landrini 2003). The fluid is treated as weakly compressible in the present approach, which allows the use of an equation of state to determine fluid pressure, rather than solving a Poisson's equation. The permissible time step for a stable computation decreases with increasing sound speed. This precludes using the actual speed of sound in water to specify the compressibility as this would result in really small time steps. It has been shown by Monaghan (1994) that the numerical solutions are not influenced by the sound waves as long as the sound speed in the fluid is greater than ten times the maximum fluid velocity. Therefore, the compressibility is artificially specified to satisfy the above condition and allows for reasonable time step size. The Beeman algorithm (Beeman 1976) is applied in the numerical simulations to update the solutions in time. This time stepping algorithm uses a Beeman predictor step and an Adams-Bashforth-Moulton corrector step and is accurate to third order. A variable time step is calculated according to Monaghan and Kos (1999).

2.3 *Coupling theory*

The hybrid model presented here is a two way coupling, this means that information is transferred between models in both directions. Preliminary approaches of a hybrid model, which involved a one way coupling where information was transferred from the Boussinesq model to the SPH model, can be found in Narayanaswamy and Dalrymple (2005) and Crespo *et al*. (2008).

Consider the problem domain shown in Fig. 1. The domain is subdivided into a Boussinesq region and a SPH region. The solution in the domain is carried out using FUNWAVE from the offshore region till the virtual boundary CD. The flow is solved using SPHysics from virtual boundary AB till the dry beach. The overlap region ABCD is positioned offshore of the breaking zone so that the Boussinesq model computes the wave transformation in the non-breaking region and the SPH model handles the breaking and runup phases of the wave field. The Boussinesq model provides the boundary condition to the SPH model at boundary AB. Feedback from the SPH model to the Boussinesq model is provided through boundary conditions at the Boussinesq boundary CD.

The Boussinesq model solves for the nodal horizontal velocity u_α, and wave height η_α at a reference depth $z = z_\alpha(x)$. At each particle i (small dots in Fig. 1), the SPH model solves for velocities (u_i, w_i), position (x_i, z_i), density ρ_i and pressure P_i.

At each Boussinesq time step, the boundary values $(u_{\alpha b}, \eta_{\alpha b})$ for FUNWAVE at the Boussinesq-SPH interface (line CD in Fig. 1), are specified using the information obtained from the SPH model. This technique to calculate the average velocity at a certain point was also applied in Gómez-Gesteira *et al.* (2005). The velocity is obtained by computing the smoothed particle velocity at elevation $z = z_\alpha(x)$ from the SPH model as

$$u_{\alpha b} = \sum_j u_j V_j W_{bj} \tag{8}$$

The summation is carried out over all particles within a radius of $2h$ from $(x_{\alpha b}, z_\alpha(x_{\alpha b}))$ where $x_{\alpha b}$ is the x-coordinate position of line CD. The boundary value of the wave-height is obtained by determining the particle with the largest z value at $x_{\alpha b}$.

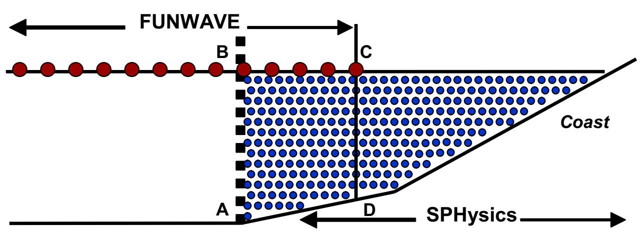

Figure 1 Schematic of the domain subdivision in the coupled model. Big dots correspond to Boussinesq model nodes, the small ones are SPH particles, the black squares are the wavemaker particles and the *ABCD* area is the overlap region

The boundary conditions for the SPH model are implemented in the form of a Boussinesq wavemaker at boundary AB. A column of SPH boundary particles is placed along AB at $t = 0$. Subsequently, the velocity of these boundary particles is determined from the velocity of the adjacent Boussinesq nodes. The position of the wavemaker particles are then computed from these velocities.

Since the SPH timestep is much smaller than the Boussinesq time step, the Boussinesq nodal velocities at each SPH time step are computed using a linear interpolation in time. The temporal grids in the SPH and Boussinesq models are staggered by $0.5\Delta t^b$ where Δt^b is the Boussinesq time step i.e. the SPH time lags the Boussinesq time by half a Boussinesq time step. When the SPH computations are carried out, the Boussinesq nodal velocities at each SPH time step are computed using a linear fit of the two most recent Boussinesq solutions.

Let the coordinates of a boundary particle P be denoted by (x_p, z_p). The Boussinesq nodes on either side of x_p are first determined at each SPH time step. The Boussinesq velocity $u_{\alpha p}$ at x_p is then determined by a linear interpolation in space. The Boussinesq model assumes a quadratic variation of the horizontal velocities over the depth. Hence, the velocity at P is determined as

$$u_p = u_{\alpha p} + \frac{\partial z_\alpha}{\partial x}\frac{\partial h^* u_\alpha}{\partial x} + (z_\alpha - z_p)\frac{\partial}{\partial x}\frac{\partial h^* u_{\alpha p}}{\partial x} + z_\alpha \frac{\partial z_\alpha}{\partial x}\frac{\partial u_{\alpha p}}{\partial x} + \frac{1}{2}(z_\alpha^2 - z_p^2)\frac{\partial}{\partial x}\frac{\partial u_{\alpha p}}{\partial x} \tag{9}$$

The position of the boundary particle P is then updated using $x_p^{n+1} = x_p^n + \Delta t^S u_p$, where Δt^s is the SPH time step.

3 Working example

To prove the capability of this hybrid approach, the model is used to simulate the propagation of a solitary wave in a constant depth tank. The solitary wave is generated in the FUNWAVE domain. The wave is propagated into the region modeled by SPHysics, it is reflected at the end wall and propagated back into the FUNWAVE domain.

Figure 2 shows the initial configuration and the subdivided domain of this testcase. The tank is 20 m long with a water depth of 0.5 m. The FUNWAVE region starts at $x = 0$ and extends to $x = 16$ m. SPHysics is used to model the wave evolution from $x = 13$ m to $x = 20$ m. The colored *ABCD* area corresponds to the overlap area. Black dots along *AB* line represent the wavemaker particles positions. These positions will change in time.

The solitary wave is generated at $x = 4$ m with an amplitude $a = 0.15$ m and a period $T_p = 1$ s. A 20 cm grid size is used on FUNWAVE with a time step of $t = 0.001$ seconds. The initial free surface elevation and velocity of the solitary wave can be obtained as a solution of the weakly nonlinear Boussinesq equations (Wei *et al.* (1995)). These are given as

$$\eta(x, t = 0) = (C_1 + C_3)\sec h^2(C_2 x) \tag{10}$$

$$u_\alpha(x, t = 0) = C_4 \sec h^2(C_2 x) \tag{11}$$

The derivation and the coefficients C_i $i = 1$–4 are described in Wei *et al.* (1995).

The SPHysics numerical parameters used in the model are summarized in Table 1.

Table 1 SPH parameters used in the numerical application

h (smoothing length)	0.0325 m
μ (kinematic viscosity)	$1e^{-6}\,m^2s^{-1}$
n_p (number of particles)	5103
n_b (boundary particles)	363
$\langle dt \rangle$ (mean time step)	$\approx 1e^{-4}$ s

Snapshots of the SPH wavemaker (line AB in Fig. 2) at different time instants are depicted in Fig. 3. Each snapshot shows the $u_\alpha = u(x, z_\alpha)$ velocity at each x-node (solid line) and wavemaker velocities (arrows). The first snapshot (time = 4 s) corresponds to the wave advancing in the x-direction, which corresponds to positive u_α. Using these values of u_α and Eq. (2), positive velocity values are obtained for SPH wavemaker particles (arrows). Hence the wavemaker advances in x-direction. The second snapshot (time = 8 s) corresponds to the reflected wave after hitting the right wall. As expected, the observed negative values of u_α generate negative velocities for wavemaker particles and the wavemaker propagates the wave offshore towards FUNWAVE region.

In order to analyze the performance of this hybrid model, results from the FUNWAVE part of the hybrid model and results

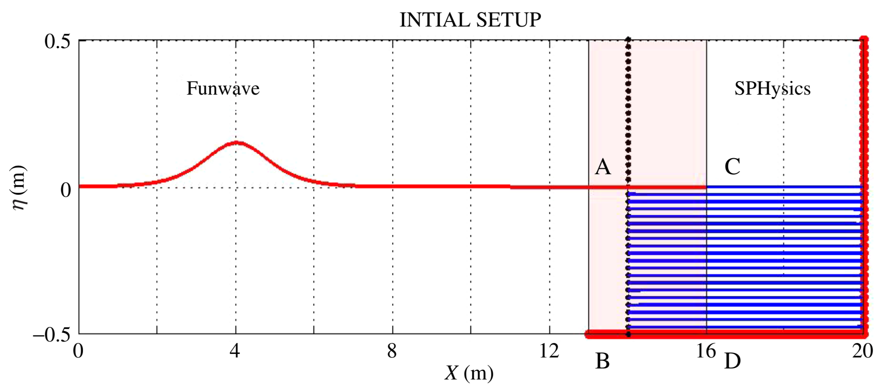

Figure 2 Initial setup of the testcase

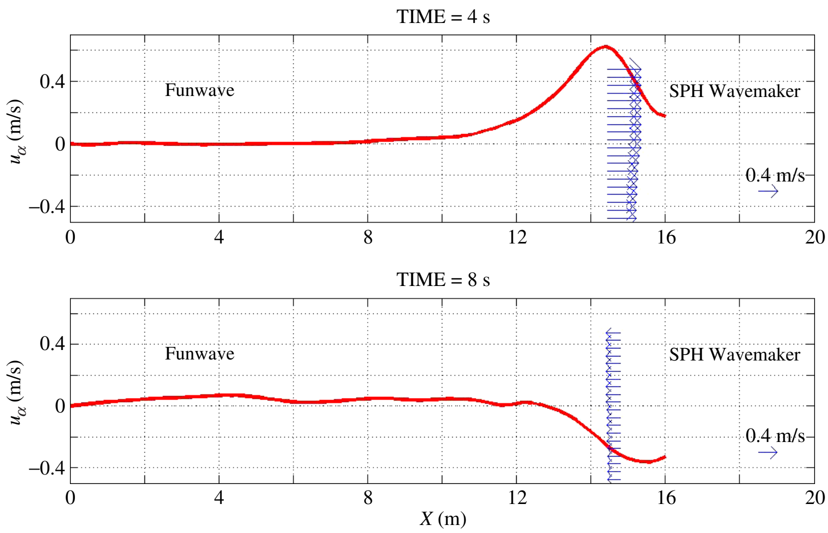

Figure 3 Wavemaker movement

obtained from using FUNWAVE to propagate the solitary wave over the entire domain are compared. Figure 4 shows the comparison between the free-surface elevation from solving the hybrid model (crosses) and from a pure FUNWAVE simulation of the solitary wave evolution in the entire domain for the simulation (solid line). The three first snapshots show the incoming wave propagating towards the SPH region and the last three figures correspond to the reflected wave. A better agreement is observed before the wave hits the end wall, while bigger differences are found observing the reflected wave. One reason for this is due to the fact that the impact mechanism of the wave against the end wall is handled by SPHysics in the hybrid model using a repulsive force and is handled in the pure FUNWAVE simulations by prescribing the reflection conditions of zero normal velocity and zero free surface slope in FUNWAVE. Also, the FUNWAVE boundary conditions are prescribed at line CD in Fig. 2 using Eq (1). This could lead to the differences observed in the wave heights as the simulation progresses.

Apart from this visual comparison, the observed differences in the snapshots of Fig. 4 can be quantified considering two statistical parameters as explained in Crespo *et al.* (2007):

$$A_r = \sqrt{\sum_{i=1}^{N} \left(F_i^*\right)^2 \Bigg/ \sum_{i=1}^{N} \left(F_i^{\mathrm{FUN}}\right)^2} \tag{12}$$

$$P_d = \sqrt{\sum_{i=1}^{N} \left(F_i^* - F_i^{\mathrm{FUN}}\right)^2 \Bigg/ \sum_{i=1}^{N} \left(F_i^{\mathrm{FUN}}\right)^2} \tag{13}$$

where F_i^* and F_i^{FUN} refer to the free-surface position obtained from the hybrid model and from the pure FUNWAVE simulation respectively.

A perfect agreement between both signals should result in $A_r \to 1$ and $P_d \to 0$. The results obtained using Eqs. (12)–(13) for the time instants depicted in Fig. 5 are shown in Table 2.

For the incident wave (Time = 3 s–5 s), the average parameters are $A_r = 0.93$ and $P_d = 0.05$, and for the reflected wave (Time = 7 s–9 s) are $A_r = 0.78$ and $P_d = 0.12$. It is observed from the statistical analysis as well that the incident waves as observed in the hybrid model and FUNWAVE compare better than the reflected wave. This test establishes the fact that the mechanism to transfer information from the SPHysics to FUNWAVE at line CD in Fig. 2 works reasonably well.

Figure 5 shows snapshots of the solitary wave at different instants in time obtained using the hybrid model. FUNWAVE results are plotted along the first 16 meters (solid line). The overlap region is represented by a colored background (13–16 m) and dots represent the SPHysics particles. The same instants of time as shown in Fig. 4 are depicted here i.e. the incoming wave is observed in the first three snapshots, the wave hits the right wall at Time = 6 s and then the last three plots show the reflected wave as it propagates towards the Boussinesq region.

From Fig. 5, it can be observed that the wave heights predicted by FUNWAVE and SPHysics compare well in the overlap region for both the incident and reflected phases of the solitary wave propagation.

Following the same procedure used to quantitatively compare the results from the hybrid model and the pure FUNWAVE simulation, statistical parameters [Eqs. (12)–(13)] are calculated to measure the accuracy of the information transfer at the boundaries between the FUNWAVE and the SPH interfaces of the hybrid model. F_i^* in Eqs. (12)–(13) now refers to the free-surface position obtained from SPH. Table 3 shows the results of the statistical analysis obtained for the time instants depicted in Fig. 5.

4 Concluding remarks

A Hybrid SPHysics-FUNWAVE model to study coastal wave propagation has been developed and presented. This model has been developed with the idea of using FUNWAVE to propagate waves, with low computational cost, accurately till the surf zone and then using SPHysics to handle the breaking and post-breaking processes as it can naturally track multiply connected free surfaces. Thus, the advantages of both models are used to develop an efficient wave model. One of the key developments achieved in

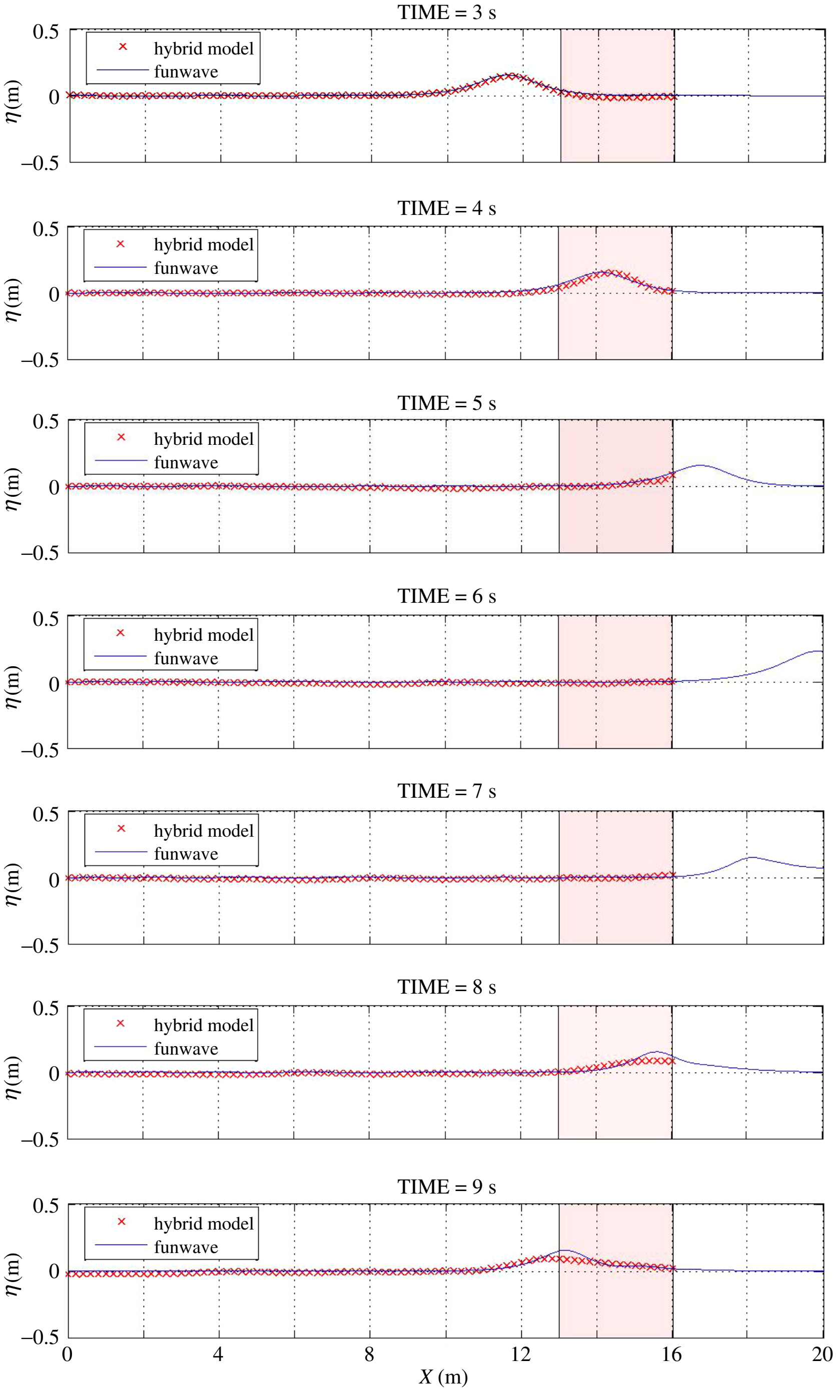

Figure 4 Comparison of FUNWAVE free-surface results

Table 2 Statistical comparison between the positions of the free-surface calculated only by FUNWAVE (solid line in Fig. 4) and by FUNWAVE coupled to SPH (crosses in Fig. 4) in the overlap region

Time	A_r	P_d
3 s	0.97	0.05
4 s	0.93	0.04
5 s	0.88	0.05
6 s	0.82	0.08
7 s	0.83	0.09
8 s	0.74	0.16
9 s	0.77	0.12

Table 3 Statistical comparison between the position of the free-surface calculated only by FUNWAVE coupled to SPH (solid line in Fig. 5) and by SPH coupled to FUNWAVE in the overlap region. The free surface was calculated according to the position of the particles shown in Fig. 5

Time	A_r	P_d
3 s	0.97	0.14
4 s	0.95	0.39
5 s	1.19	0.43
6 s	1.14	0.26
7 s	1.11	0.23
8 s	1.15	0.25
9 s	0.90	0.21

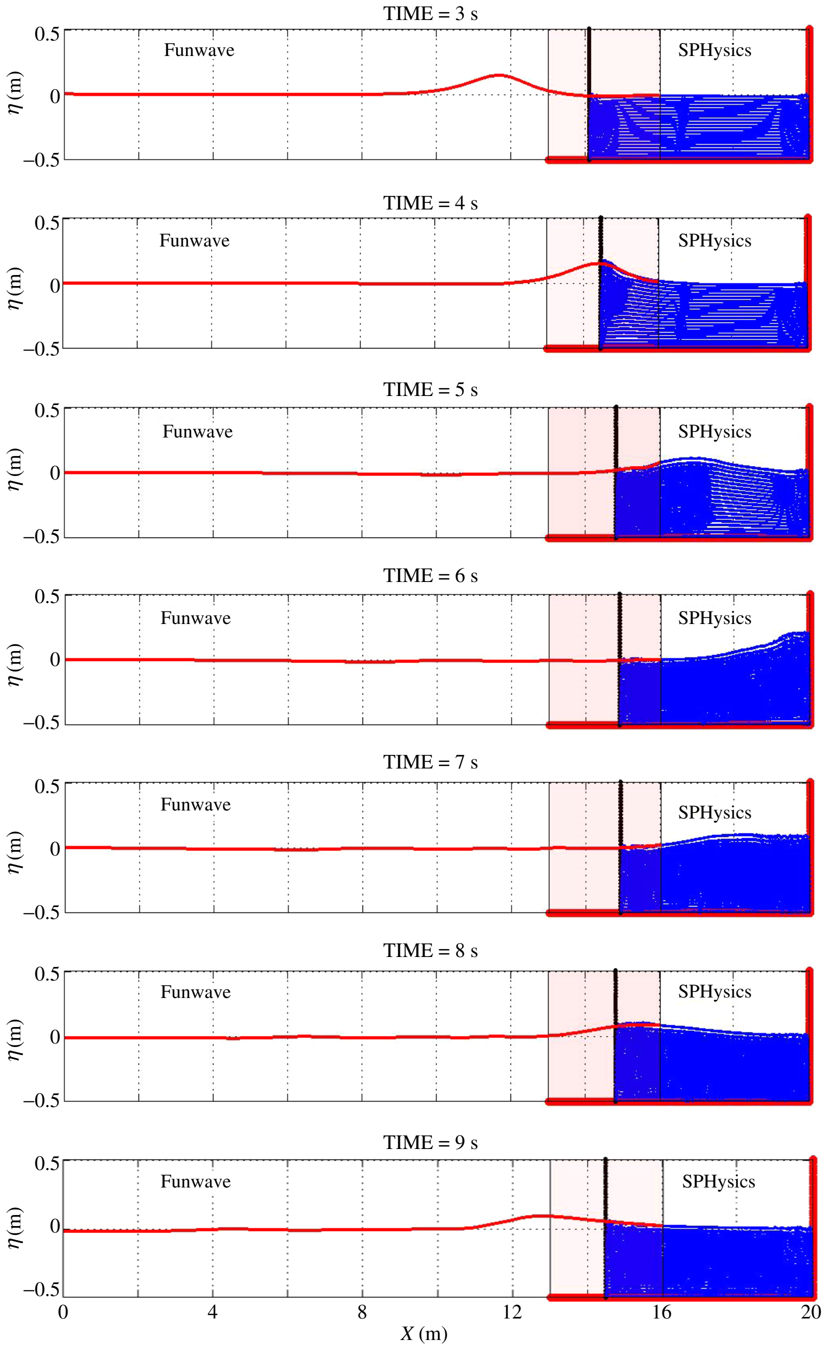

Figure 5 Different instants of wave propagation using the hybrid model

this model is the algorithm to prescribe the boundary conditions for the individual models in the overlap region.

A simple working case has been used to demonstrate the capability of the model to propagate a solitary wave in a constant depth tank. The results of the hybrid model were first compared with a pure FUNWAVE simulation of the same solitary wave in the same tank. Good free surface comparisons were found for the incident wave, although some differences were obtained for the reflected wave. These differences are due the formulation of viscous effects, and the manner in which wave reflection is treated in the individual models.

Subsequently, different the performance of the hybrid model has been assessed by comparing the FUNWAVE and SPHysics results of the wave heights in the overlap region at instants of time. Good comparisons between the two results were observed demonstrating the ability of the model to accurately transfer information between the SPHysics and FUNWAVE regions.

Additional research should be conducted to validate this hybrid model using published experimental data of solitary wave runup on planar beaches. Special attention should be paid to the study of the conservation properties of this coupling algorithm. Specifically, it is not clear that mass is conserved due to the manner in which information from SPH is used to specify the boundary conditions for the Boussinesq model. Once this model has been validated and the conservation properties studied, it is envisioned that a similar procedure will be carried out to couple 2D-FUNWAVE and 3D-SPHysics.

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Notation

a = Incident wave amplitude
A_r = Relative amplitude (statistical parameter)
C_1, C_2, C_3 = Parameters in Boussinesq equation
g = Gravity acceleration
h = Smoothing length
h^* = Reference water depth
k = Reference wave number
n_p = Number of particles
P_d = Phase difference (statistical parameter)
T_p = Period of the incident wave
u_i, w_i = Velocity of particle i
u_α = Nodal Boussinesq solution
x_i, z_i = Position of particle i
z_α = Arbitrary reference where nodal FUNWAVE velocities are evaluated
Δt^b = Boussinesq time step
Δt^S = SPHysics time step
ε = Dimensionless parameter
η = Free-surface elevation
μ = Kinematic viscosity
μ^* = Dimensionless parameter
ρ_i = Particle density

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