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A Unified SPH Framework for Flexible Fluid-Structure Interaction with DualSPHysics

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Overview

- Background & Motivation
- Smoothed Particle Hydrodynamics
- Structural Modelling with SPH
- Fluid-Structure Coupling
- Validation & Results
- Progress This Week

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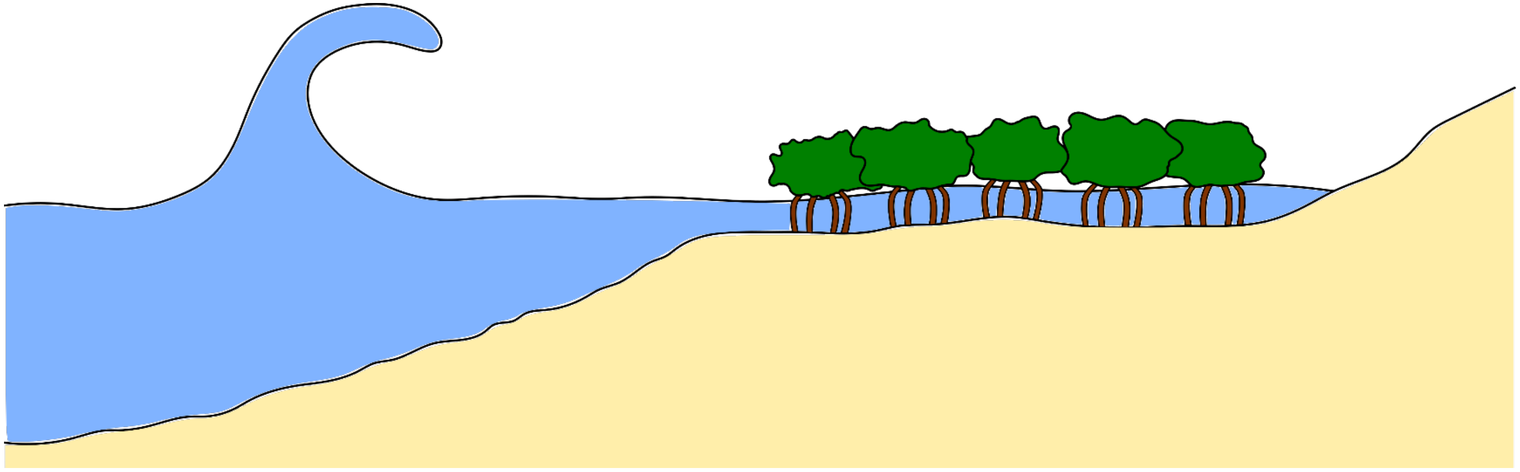
Coastal Vegetation

- Coastal vegetation has been widely promoted as a cost-effective barrier to coastal inundation due to tsunamis/storm surges
- This has led to extensive reforestation initiatives – however, need to understand best approaches for designing these ‘bioshields’



Coastal vegetation provides a mechanism for protection from tsunamis and storm surges.

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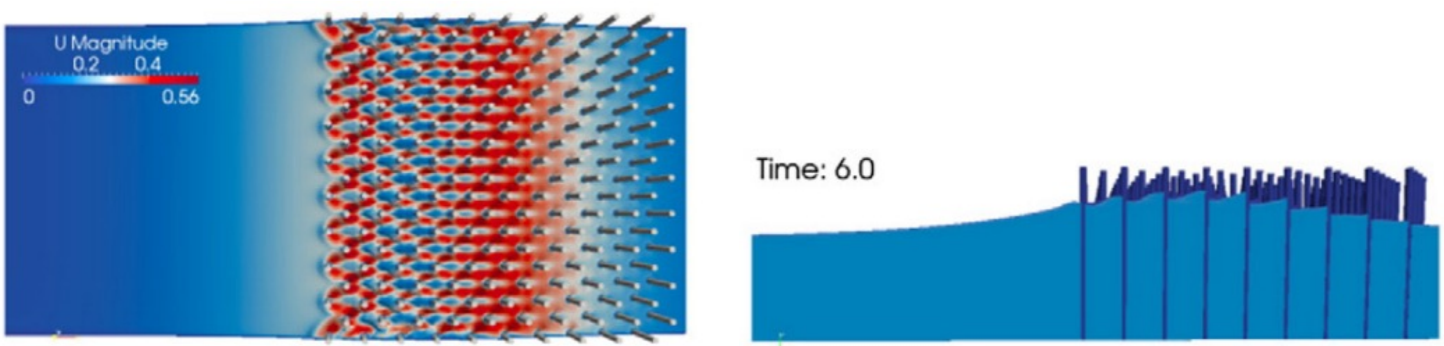
Modelling Coastal Vegetation

- Typical approach to modelling coastal vegetation is some form of reduced-order modelling with simplifying assumptions
  - For example, using a porous layer model (bulk drag coefficient) or rigid vegetation
- Some recent studies have suggested that this can lead to under-predicting flow forces and over-predicting wave attenuation
- Opportunity for higher fidelity modelling with hardware acceleration (GPUs)



Flow velocity and surface elevation of wave through array of rigid emergent cylinders (Maza et al. 2015).

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Governing Equations

- The governing equations for (Lagrangian) weakly-compressible SPH are:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \nabla \cdot \mathbf{u}$$

Conservation of mass

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma} + \mathbf{g}$$

Conservation of momentum

- For fluids, the Cauchy stress is split into an isotropic and deviatoric part:

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g}$$

- The mass and momentum equations are coupled via an equation of state

- The SPH discretisation provides operators for the derivatives in the equations

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Structural Modelling with SPH

- Opted for an SPH-based approach to model the structure:
  - Easier integration within DualSPHysics
  - Monolithic / unified schemes provide enhanced stability over partitioned approaches
  - Better suited to modelling additional complex processes (e.g. fracture)

- Momentum equation for a continuum:

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{1}{\rho}\nabla \cdot \boldsymbol{\sigma} + \mathbf{g}$$

- Can split stress tensor into an isotropic and deviatoric part and solve just like a fluid (with different state equation and constitutive model)

- As it is, there are three problems with this approach: 1) tensile instability; 2) linear inconsistency; 3) rank deficiency / hourglassing

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Tensile Instability

- Solution is to adopt a Total Lagrangian approach (Belytschko et al. 2000, Rabczuk et al. 2004)

- Reformulate momentum equation with respect to a reference (initial) configuration:

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{1}{\rho_0}\nabla_0 \cdot \mathbf{P} + \mathbf{g}$$

- Cauchy stress tensor is replaced with nominal (first Piola-Kirchoff) stress tensor and standard SPH discretisation is applied

- Everything is measured with respect to initial configuration:
  - No need to recompute kernel derivatives
  - No need to recompute neighbouring particles
  - No need to track ‘hydrodynamic’ quantities (density, pressure etc.)
  - No need to compute continuity equation

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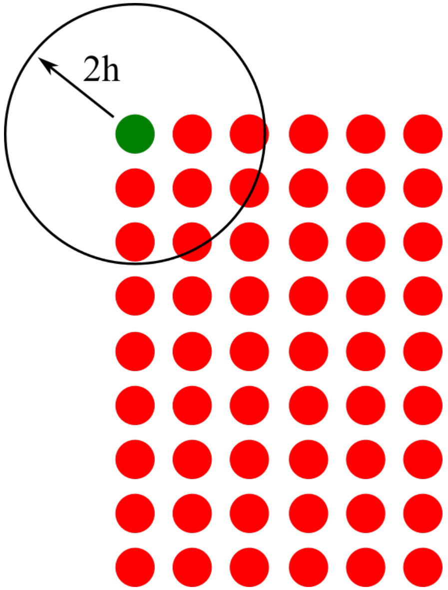
Linear Inconsistency

- Boundaries are a big problem for structural dynamics with SPH due to incomplete support
- Need to reproduce gradient of a linear field (Randles & Libersky 1996)
- Introduce a kernel correction:

$$\tilde{\nabla}_a W_{ab} = \mathbf{L}_a^{-1} \nabla_a W_{ab}$$

$$\mathbf{L}_a = \sum_b \frac{m_b}{\rho_b} \mathbf{x}_{ba} \otimes \nabla_a W_{ab}$$



Particles near edge do not have full support within kernel radius.

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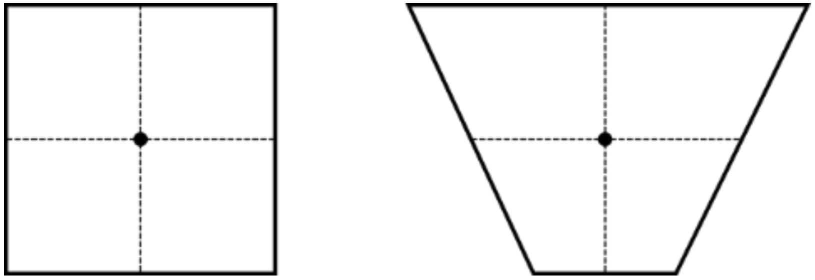
Rank Deficiency / Hourglassing

- Rank-deficiency leads to zero-energy modes which are not suppressed and eventually become unstable (similar to reduced order elements in FEM)

- Options for suppressing these modes are:
  - Stress integration points
  - Reformulate into mixed-base set
  - Corrective force



Reduced order elements cannot capture certain deformation modes.

- For the corrective force approach you penalise any deformation which is not described exactly by the deformation gradient (Ganzenmuller 2015)

- Easy to implement and efficient however it modifies the effective stiffness and introduces a tuning parameter

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Discretisation and Material Model

- Finally, the discrete form of the momentum equation of the structure is:

$$\frac{\mathrm{D}\mathbf{u}_a}{\mathrm{D}t} = \sum_b m_{0b} \left(\frac{\mathbf{P}_a \mathbf{L}_{0a}^{-1}}{\rho_{0a}^2} + \frac{\mathbf{P}_b \mathbf{L}_{0b}^{-1}}{\rho_{0b}^2} \right) \cdot \nabla_{0a} W_{0ab} + \frac{\mathbf{f}_a^{HG}}{m_{0a}} + \mathbf{g}$$

- The first Piola-Kirchhoff stress is related to the second Piola-Kirchhoff stress:

$$\mathbf{P} = \mathbf{F}\mathbf{S}$$

- The second Piola-Kirchhoff stress is related to the Green-Lagrange strain via the Saint Venant-Kirchhoff constitutive model:

$$\mathbf{S} = \lambda \mathrm{tr}(\mathbf{E})\mathbf{I} + 2\mu\mathbf{E}$$

- Where the Green-Lagrange strain and deformation gradient are given by:

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right) \quad \text{and} \quad \mathbf{F} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{x}_0}$$

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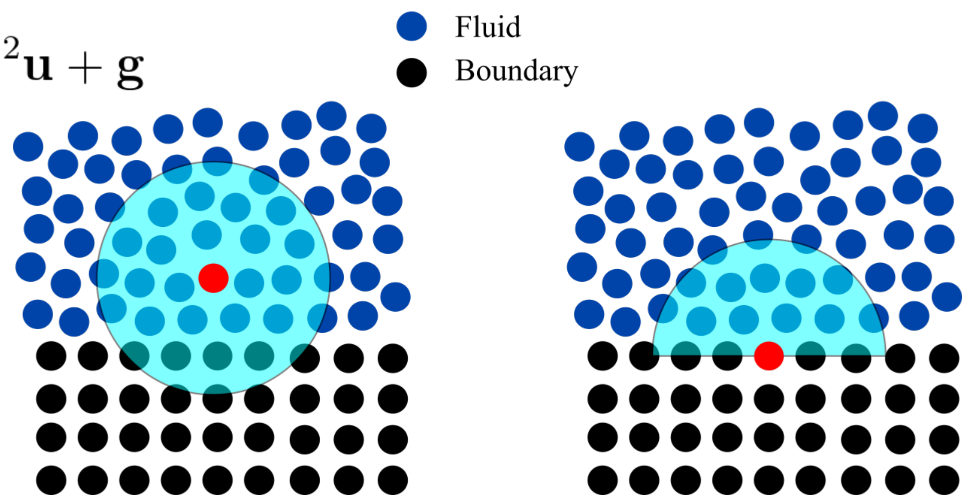
Dynamic Boundary Condition

- The dynamic boundary condition is the basic pre-existing boundary condition within DualSPHysics
- Density of boundary particles is evolved via continuity equation as normal
- Momentum equation is not computed for boundary particles

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g}$$

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \nabla \cdot \mathbf{u}$$



$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \nabla \cdot \mathbf{u}$$

Kernel stencil for fluid (left) and boundary (right) particle.

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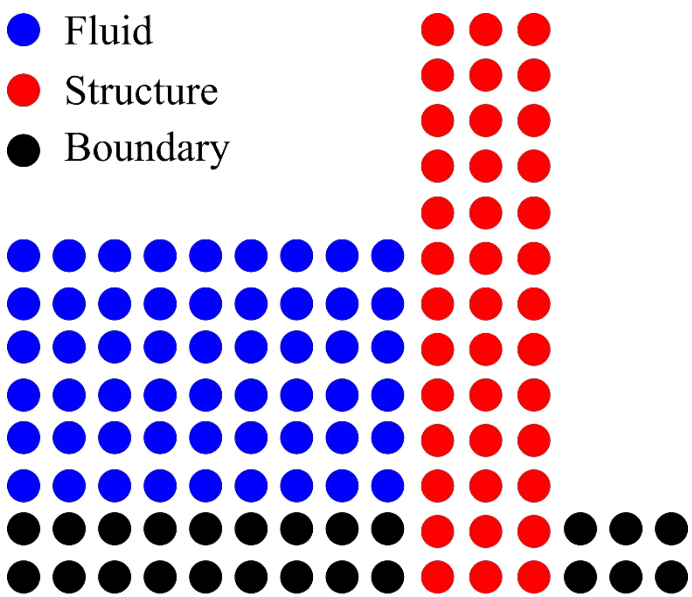
Fluid-Structure Coupling

- The fluid-structure coupling is handled via the same approach (dynamic boundary condition)
- Fluid see structural particles as normal boundary particles (with a velocity)
- Structure sees fluid particles in the same way a boundary particle does
- Momentum equation is integrated for structure particles but not for boundary
- No need to know geometric information about interface (e.g. surface normals)



Particle types used for fluid-structure coupling.

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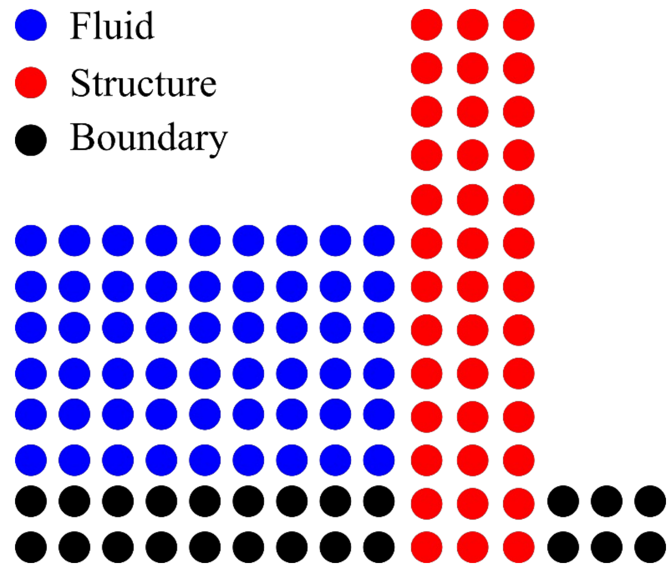
Fluid-Structure Coupling

- Total force on a particle is sum of contributions from neighbouring fluid, structure and boundary particles
- Note that the last two terms in structure equation use the Total Lagrangian form



Particle types used for fluid-structure coupling.

Fluid Particle

$$\frac{\mathbf{D}\mathbf{u}_a}{\mathrm{D}t} = -\sum_b m_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2}\right) \nabla_a W_{ab} - \sum_b m_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2}\right) \nabla_a W_{ab} - \sum_b m_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2}\right) \nabla_a W_{ab}$$

Structure Particle

$$\frac{\mathbf{D}\mathbf{u}_a}{\mathrm{D}t} = -\sum_b m_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2}\right) \nabla_a W_{ab} + \sum_{b0} m_{0b} \left(\frac{\mathbf{P}_a}{\rho_{0a}^2} + \frac{\mathbf{P}_b}{\rho_{0b}^2}\right) \cdot \tilde{\nabla}_{0a} W_{0ab} + \sum_{b0} m_{0b} \left(\frac{\mathbf{P}_a}{\rho_{0a}^2} + \frac{\mathbf{P}_b}{\rho_{0b}^2}\right) \cdot \tilde{\nabla}_{0a} W_{0ab}$$

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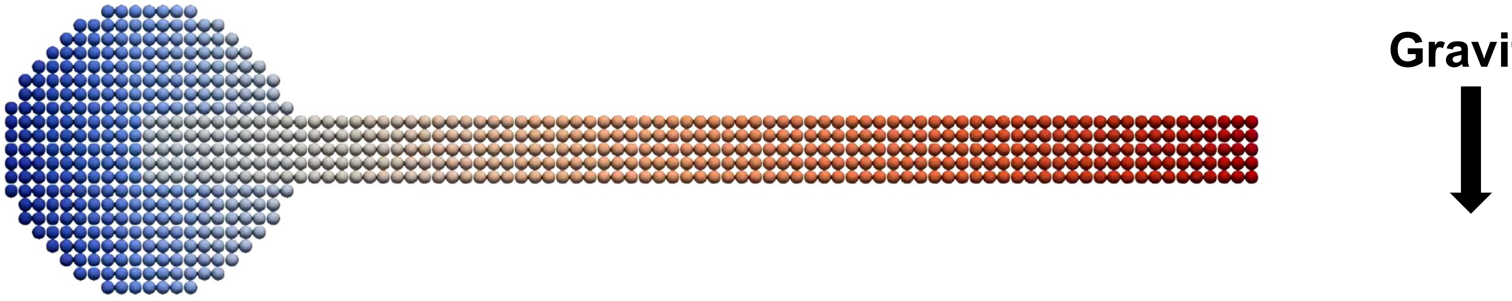
Structural Model Validation

- The structural model is first tested on its own against a popular benchmark case (Turek & Hron 2006)

- The case is a clamped beam oscillating under its own weight (no damping)



Animation of structural model validation. Particles coloured by particle ID.

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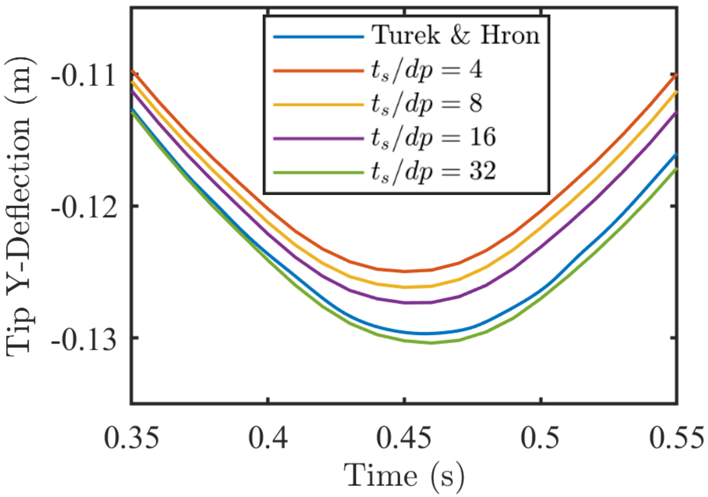
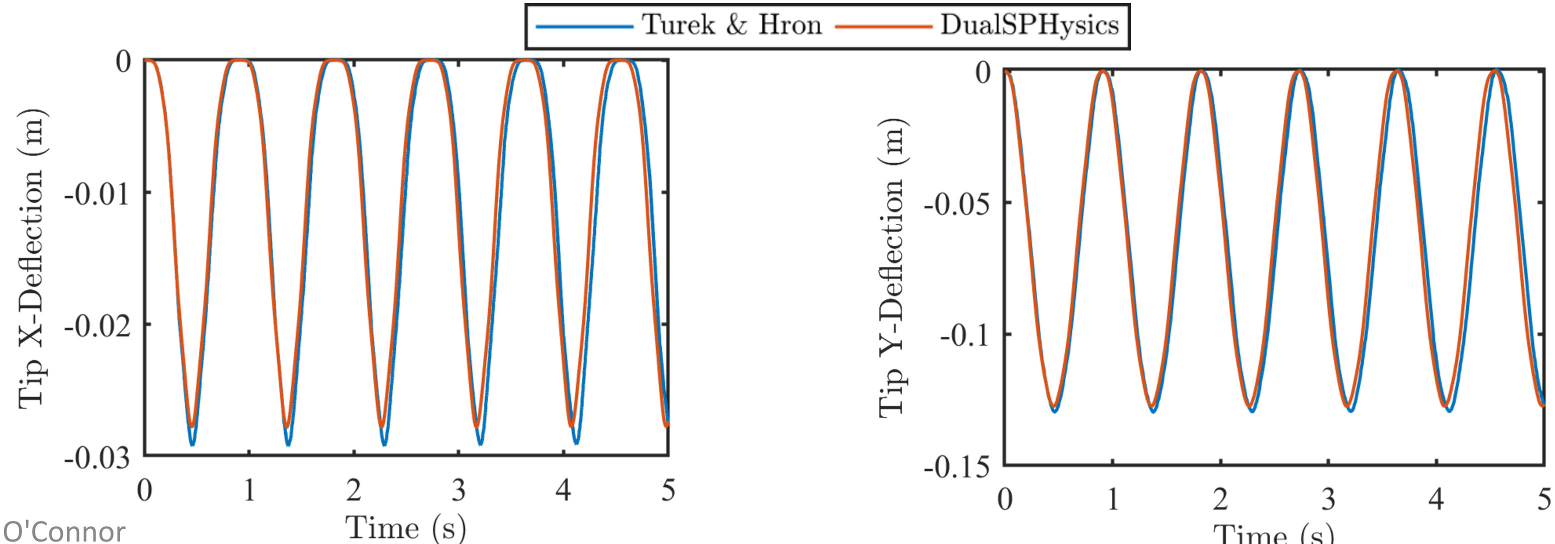
Structural Model Validation

- Tip deflections agree very well with benchmark data (FEM)
- Converges towards benchmark solution with increasing resolution
- t/dp = 4 is minimum required



Particle resolution study.



Tip deflection history compared against benchmark (t/dp = 16).

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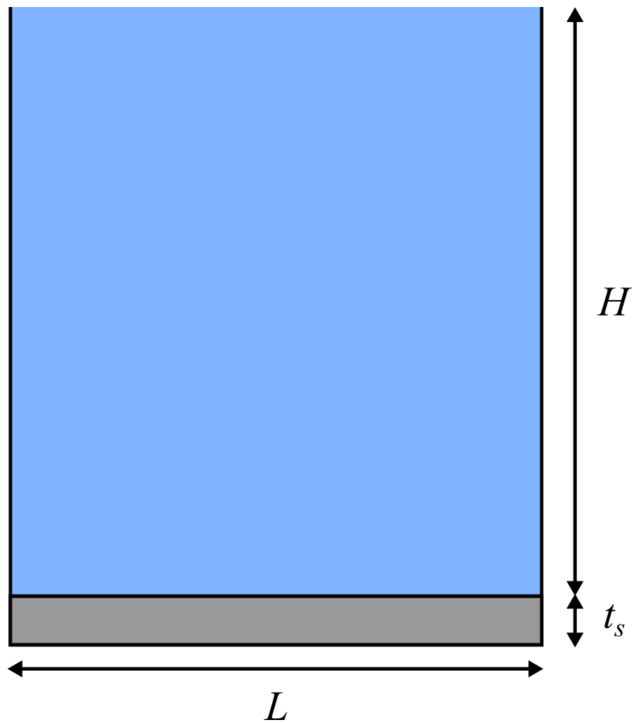
FSI Validation – Hydrostatic

- Hydrostatic water column on an initially undeformed elastic plate
- Plate deflection oscillates around equilibrium solution (with / without damping)
- Equilibrium deflection has analytical solution for sufficiently small deflection
- A range of particle resolutions are tested from t/dp = 5 to t/dp = 40
- Tests are also performed with and without delta-SPH / density diffusion (Molteni & Colagrossi 2009)



Schematic of hydrostatic case.

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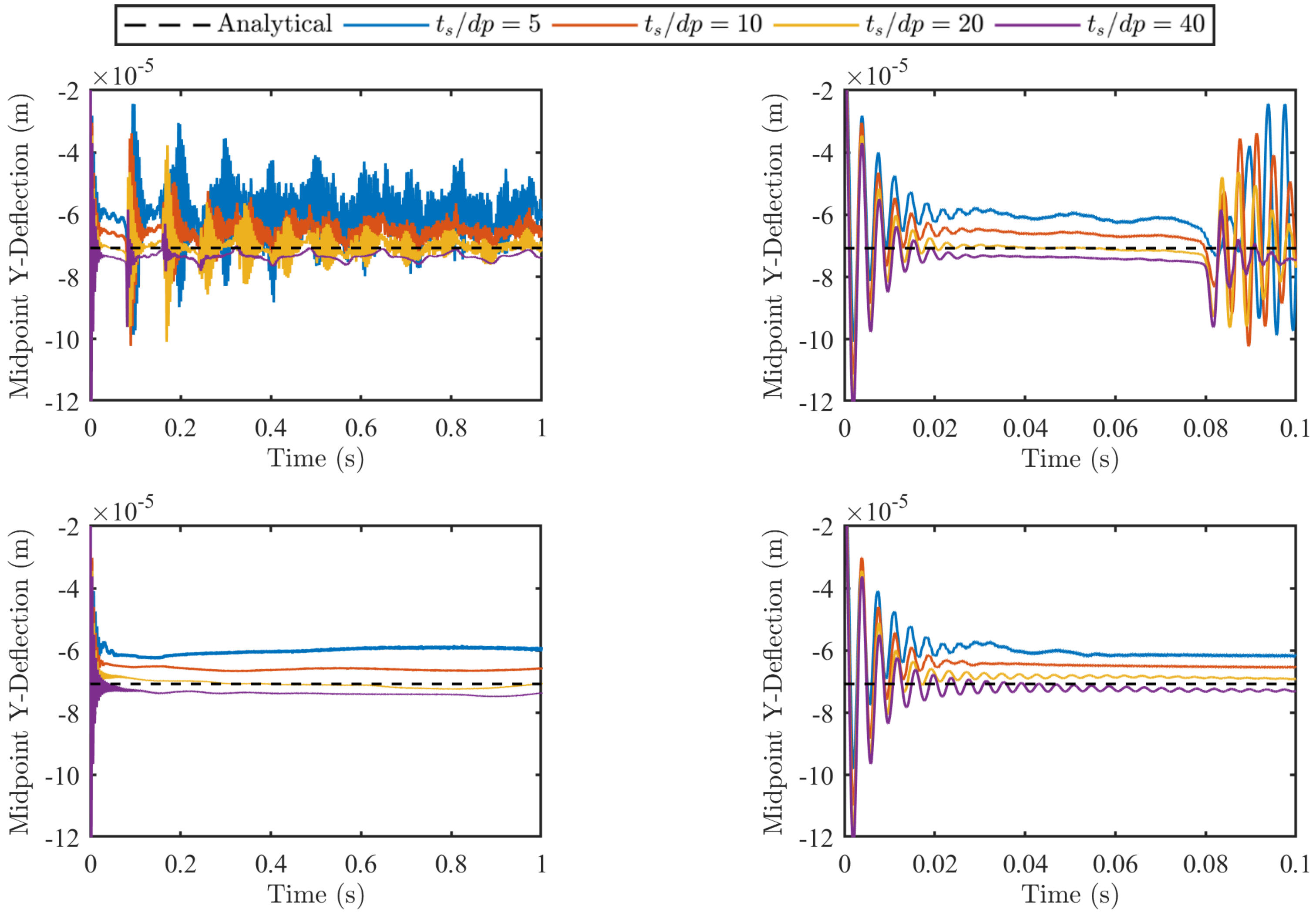
FSI Validation – Hydrostatic



Midpoint deflection of elastic beam without delta-SPH (top) and with delta-SPH (bottom).

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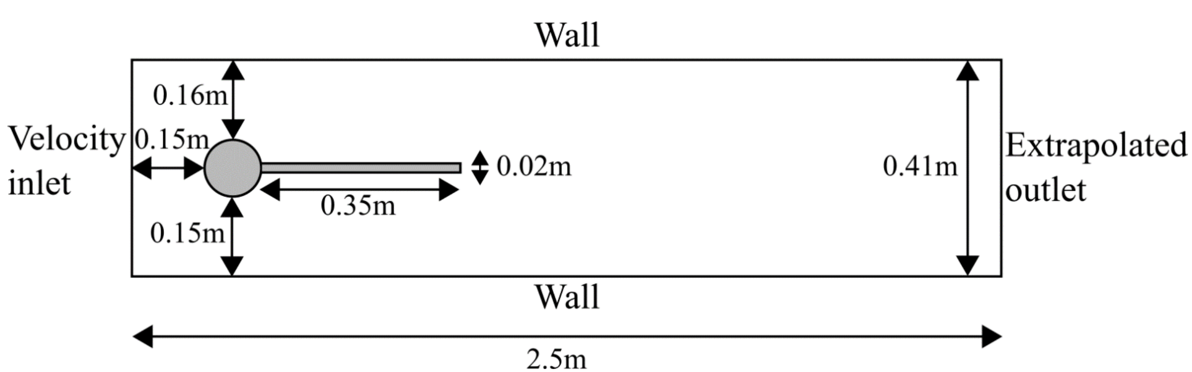
FSI Validation – Flapping Beam

- Rigid cylinder with attached flexible beam at Re = 100 (flapping motion)
- Simulation setup:
  - Inlet/outlet boundary conditions (Tafuni et al. 2018)
  - Laminar viscosity (Morris et al. 1997, Lo & Shao 2002)
  - Particle shifting (Lind et al. 2012)
  - Delta-SPH (Molteni & Colagrossi 2009)
  - t/dp = 16 (approximately 670,000 particles)
- 27 hours for 30s on Tesla V100 (projected ~7 weeks on 12-core CPU)



Schematic of flapping beam case (Turek & Hron 2006).

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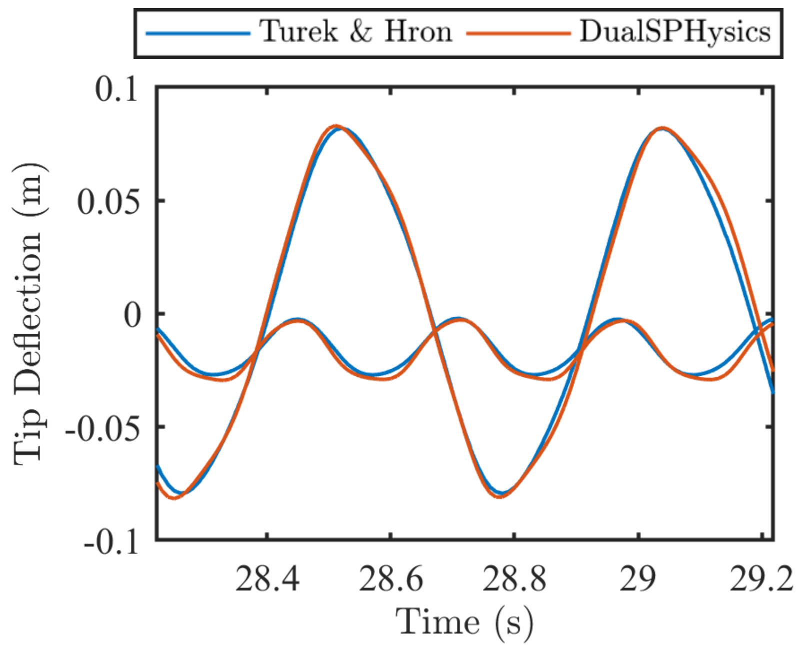
FSI Validation – Flapping Beam

- Benchmark solution is calculated via a fully implicit monolithic FEM solver with an ALE formulation
- Tip deflection agrees very well with benchmark data



Animation and tip deflection for flapping beam case (Turek & Hron 2006). Particles coloured by velocity magnitude.

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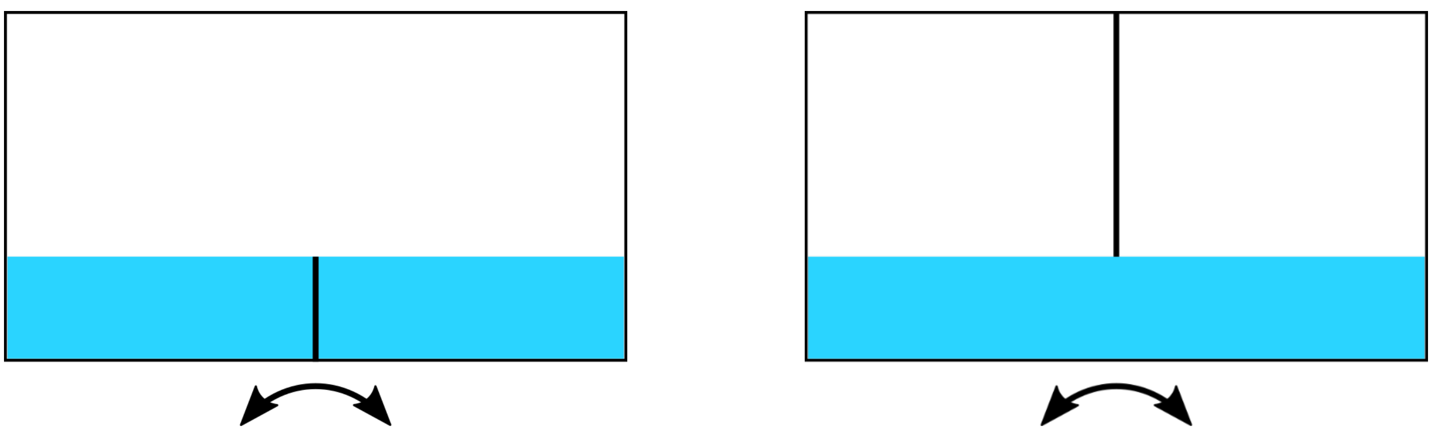
FSI Validation – Rolling Tank

- Rolling tank with a flexible beam (submerged and hanging)
- Natural frequencies are matched (submerged) and misaligned (hanging)
- t/dp = 8 (approximately 300,000 particles)
- 9 hours for 5s on Tesla V100 (projected ~2 weeks on 12-core CPU)



Schematic of rolling tank case with submerged (left) and hanging (right) beam (Idelsohn et al. 2008)

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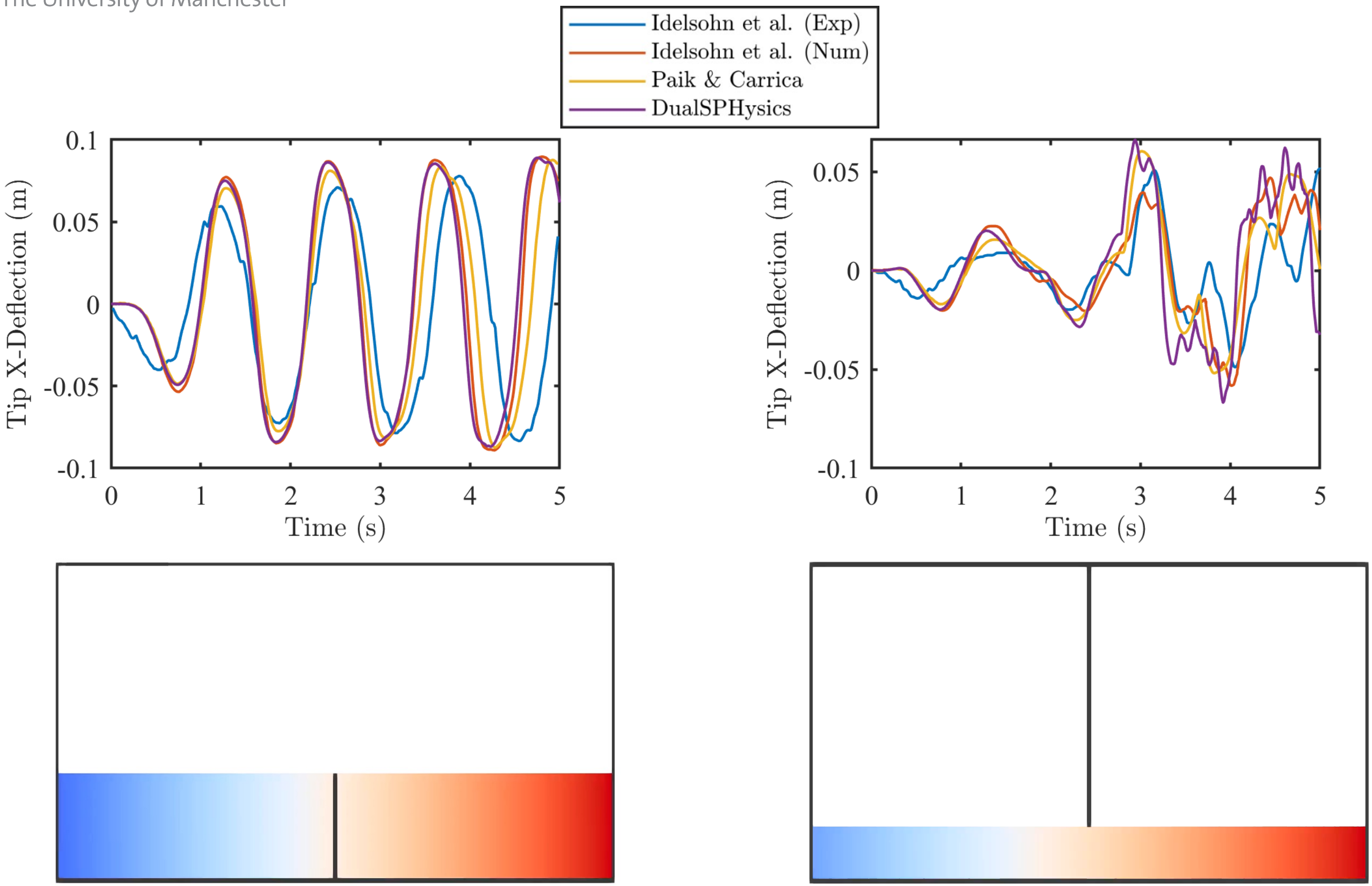
FSI Validation – Rolling Tank



Animation and tip deflection for rolling tank case (Idelsohn et al. 2008).
Particles coloured by particle ID.

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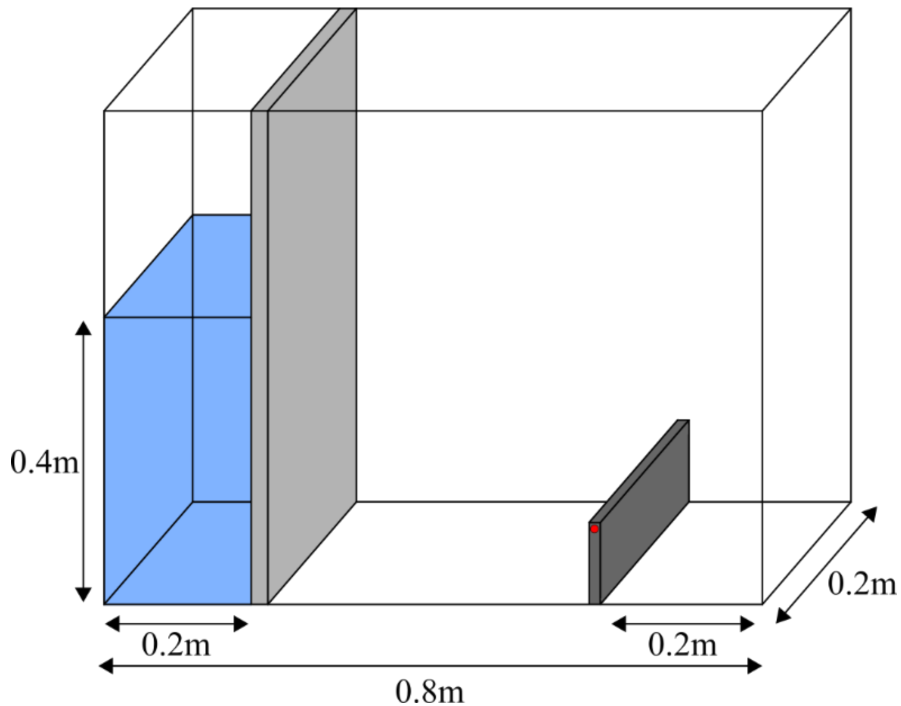
FSI Validation – 3D Dam Break

- 3D dam break impacting an elastic plate (single phase)
- First time attempted in 3D
- t/dp = 4 (approximately 25,000,000 particles)
- The gap between the sidewalls and the edge of the plate is not resolved
- 8 days for 1s on Tesla V100 (projected ~1 year on 12-core CPU)



Schematic of dam break case (Liao et al. 2015).

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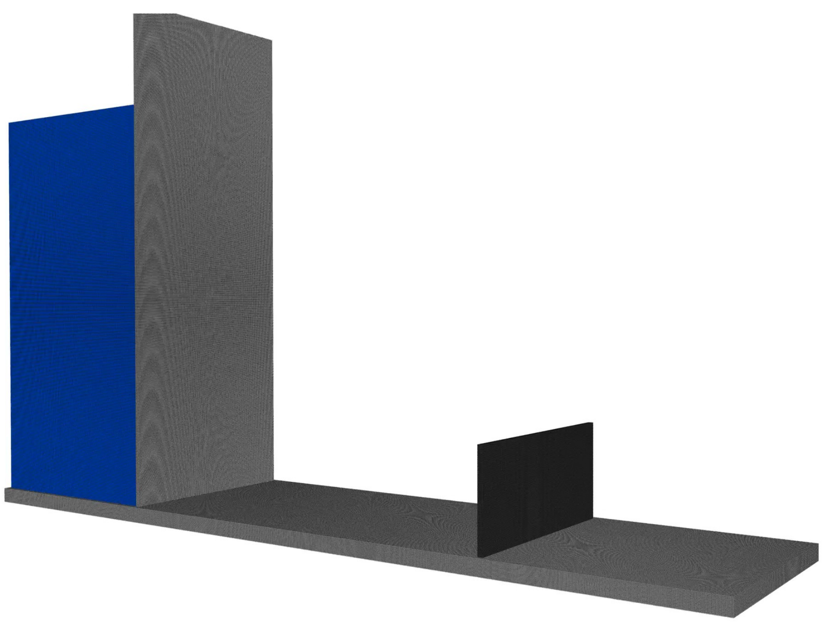
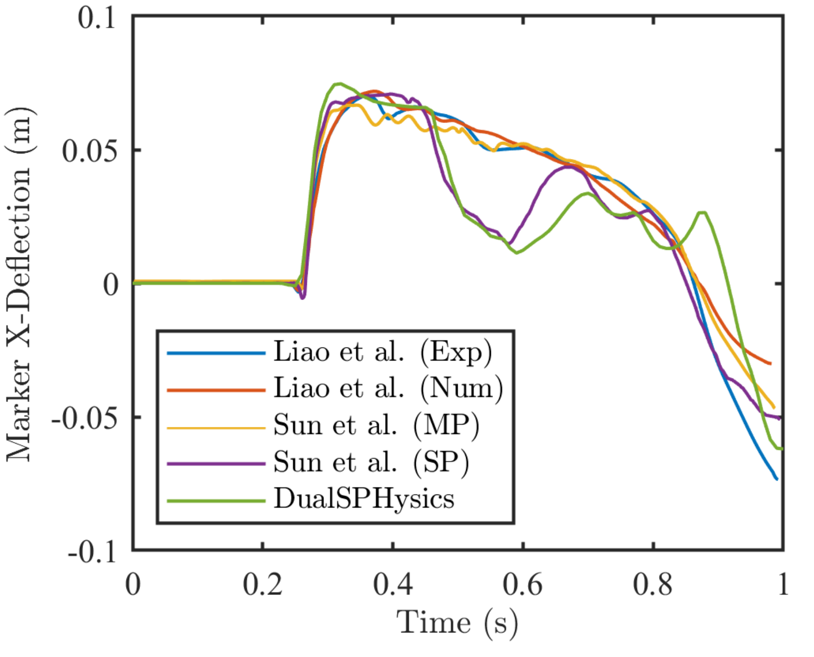
FSI Validation – 3D Dam Break

- Comparison with 2D results in literature shows reasonable agreement
- However, the single-phase (SP) vs multiphase (MP) comparison shows that it is important to correctly model the air entrainment





Animation and tip deflection for the dam break case (Liao et al. 2015). Particles coloured by velocity magnitude.

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Progress This Week

- The purpose of this week has been to reimplement this model in latest DualSPHysics version and prepare for including it in an official release

- Progress so far:
  - Moving implementation from v4.3 to v5.2 (is now working on GPU with flexible FSI)
  - Improvements to original implementation
  - Preparing example input files
  - Bug fixes!

- Still to do:
  - CPU version
  - Further improvements to implementation
  - Documentation/guides/Wiki
  - Videos

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