

A brief description of the particle finite element method (PFEM2). Extensions to free surface flows.

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Motivations for using PFEM.

- Combines convective particle movement and fixed mesh resolution.
- Accurate description of free surface flows.

Two novel features:

- Larger time steps compared to other similar numerical tools.
- Improved versions of discontinuous and continuous enriched basis functions for the pressure field.

Objectives

- Shorter computational times while the solution accuracy is maintained.
- Free surface should be reconstructed without artificial diffusion or undesired numerical effects.
- Validation of free surface flows.
 - 2D and 3D cases.
 - Low and high Reynolds numbers.

Compared to well validated numerical alternatives and experimental measurements.

From PFEM (Idelsohn-2004) to PFEM2

- Robust method where Lagrangian particles and meshing processes are alternated.
- FEM structure supports the differential equation solvers.
- Non-material points transport fixed intensive properties of the fluid.

Current *Fixed Mesh*-PFEM-2:

- X-IVAS (eXplicit Integration following the Velocity and Acceleration Streamlines).
- The initial background mesh is preserved, avoiding remeshing at each time-step.
- Mesh nodes and moving particles interchange information through interpolation algorithms.

PFEM Algorithm.

$$\begin{aligned}\nabla \cdot \mathbf{v} &= 0 \\ \rho \frac{D\mathbf{v}}{Dt} &= -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}\end{aligned}$$

Velocity and pressure are decoupled by a **fractional step method**.

- ① Predictor.
 - ① Acceleration calculation at t^n \mathbf{a}_j^n on the mesh nodes.
 - ② The X-IVAS stage to convect the fluid properties using the particles.
 - ③ The projection of the particle data to the mesh nodes.
 - ④ The implicit calculation of the diffusion term.
- ② Poisson equation.
- ③ Corrector.

Predictor: Acceleration step.

It is assumed that all fluid variables are known at time t_n for both the particles and the mesh nodes.

$$\int_{\Omega} \mathbf{a}_{\tau}^n \psi_j \, d\Omega = \int_{\Omega} \mu \nabla^2 \mathbf{v}^n \psi_j \, d\Omega = - \int_{\Omega} \mu \nabla \mathbf{v}^n \nabla \psi_j \, d\Omega + \int_{\Gamma} \mu \nabla \mathbf{v}^n \psi_j \cdot \mathbf{n} \, d\Gamma$$

$$\int_{\Omega} \mathbf{a}_p^n \psi_j \, d\Omega = - \int_{\Omega} \nabla p^n \psi_j \, d\Omega$$

$$\mathbf{a}^n = \mathbf{a}_p^n + (1 - \theta) \mathbf{a}_{\tau}^n$$

- Where θ is a numerical parameter that rules the explicitness of the viscous term in the algorithm.
- Lamping the mass matrix is often used for both problems.

Predictor: X-IVAS stage.

Evaluates the new particle position \mathbf{x}_p^{n+1} and intermediate velocity $\widehat{\mathbf{v}}_p^{n+1}$ following the velocity streamlines at t^n

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \int_n^{n+1} \mathbf{v}^n(\mathbf{x}_p^t) dt$$

$$\widehat{\mathbf{v}}_p^{n+1} = \mathbf{v}_p^n + \int_n^{n+1} [\mathbf{a}^n(\mathbf{x}_p^t) + \mathbf{f}^t(\mathbf{x}_p^t)] dt$$

N sub-stepping integration, where $N\delta t = \Delta t$

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \sum_{i=1}^N \mathbf{v}^n(\mathbf{x}_p^{n+\frac{i}{N}}) \delta t$$

$$\widehat{\mathbf{v}}_p^{n+1} = \mathbf{v}_p^n + \sum_{i=1}^N \left[\mathbf{a}^n(\mathbf{x}_p^{n+\frac{i}{N}}) + \mathbf{f}^n(\mathbf{x}_p^{n+\frac{i}{N}}) \right] \delta t$$

Predictor: Projection stage.

Projects velocity from the particles onto the mesh nodes:

$$\widehat{\mathbf{v}}_j^{n+1} = \frac{\sum_p \widehat{\mathbf{v}}_p^{n+1} W(\mathbf{x}_j - \mathbf{x}_p^{n+1})}{\sum_p W(\mathbf{x}_j - \mathbf{x}_p^{n+1})}$$

Where the Wendland kernel function was used for the projections.

Predictor: Implicit Viscosity Stage.

Implicit correction of the viscous diffusion. The fractional velocity $\widehat{\mathbf{v}}_j^{n+1}$ is found on the mesh nodes.

$$\int_{\Omega} \widehat{\mathbf{v}}_j^{n+1} \psi_j \, d\Omega = \int_{\Omega} \widehat{\mathbf{v}}_j^{n+1} \psi_j \, d\Omega + \theta \Delta t \int_{\Omega} \mu \nabla^2 \widehat{\mathbf{v}}_j^{n+1} \psi_j \, d\Omega$$

Poisson Stage.

Computes the pressure correction δp^{n+1} on the mesh nodes by solving the Poisson equation.

$$\int_{\Omega} \nabla \cdot \left[\frac{\Delta t}{\rho} \nabla (\delta p^{n+1}) \right] \phi_j \, d\Omega = \int_{\Omega} \nabla \cdot \hat{\mathbf{v}}_j^{n+1} \phi_j \, d\Omega$$

$$\frac{\partial \delta p^{n+1}}{\partial n} = 0 \quad \text{in } \Gamma_D$$

$$\delta p^{n+1} = 0 \quad \text{in } \Gamma_N$$

Pressure at time t_{n+1} is updated as $p^{n+1} = p^n + \delta p^{n+1}$.

Correction Stage.

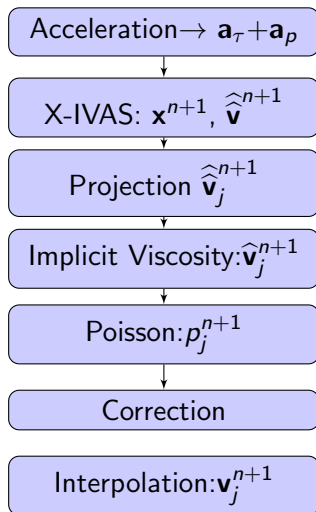
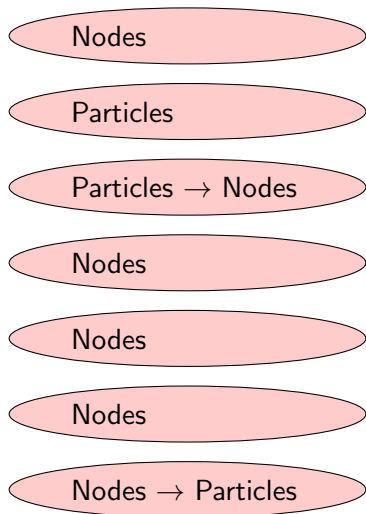
Updates the mesh and particle velocity with the pressure and diffusion corrections:

$$\int_{\Omega} \rho_j \mathbf{v}_j^{n+1} \psi_j \, d\Omega = \int_{\Omega} \rho_j \widehat{\mathbf{v}}_j^{n+1} \psi_j \, d\Omega - \Delta t \int_{\Omega} \nabla \delta p^{n+1} \psi_j \, d\Omega$$

$$\rho_p \mathbf{v}_p^{n+1} = \rho_p \widehat{\mathbf{v}}_p^{n+1} + \sum_j \delta \mathbf{v}_j^{n+1} \psi_j(\mathbf{x}_p^{n+1})$$

where $\delta \mathbf{v}_j^{n+1} = \mathbf{v}_j^{n+1} - \widehat{\mathbf{v}}_j^{n+1}$.

Summary: PFEM2 Algorithm.



Free surface flows.

- Two different fluids separated by an interface are considered
- It is essential that the interface remains sharp.
- Large jumps of fluid density and viscosity across the interface.
- Each particle p carries the information of the fluid to which it was initially assigned λ_p .

$$\lambda = \begin{cases} -1 & \text{Fluid A} \\ 1 & \text{Fluid B} \end{cases}$$

- This value is advected $\frac{D\lambda}{Dt} = 0$, i.e. each particle keeps its λ value.
- λ is projected to the mesh nodes. $\lambda_j \neq \lambda_p = \pm 1$ that the particles transport.
- Free-surface interface $\lambda = 0$

Free surface flows: velocity of the moving particles.

Two situations:

- ① All the nodes of the hosting element have the same density as the fluid particle \Rightarrow typical finite element interpolation.
- ② One or more nodes have a different density than the fluid particle (close to the interface). If $\rho_1/\rho_2 > \alpha$, two situations can appear:
 - $\rho_p = \rho_2$ (light particle tracking). The velocity used in the particle movement will be interpolated.
 - $\rho_p = \rho_1$ (heavy particle tracking). Depending on the value of

$$A = \sum_{i \in (\rho_i = \rho_p)} \psi_i$$

where the sum is limited to the hosting nodes that have the same density as the particle, we can have 2 possibilities:

Free surface flows.

- $A < \beta$ the gravity force will be included in the computation of the particle trajectory, which will finally be computed as a parabolic motion.
- $A > \beta$, the interpolation is restricted to the hosting nodes i that have the same density as the particle $\rho_i = \rho_p$.

Free surface flows. Enriched basis functions.

- We are trying to reproduce a continuous function such as *pressure* but with discontinuous derivative.
- Normally FEM basis functions are continuous, consequently a **enriched** space must be used.

Conclusion.

Life is too short, other conclusions are just dummy ideas.