Why don't we do it on the lattice

From particles to lattice and back

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Iberian SPH Meeting 2015

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Motivation

Summary

- In CFD, particle-based methods take care of convection
- The price to pay is that a mesh is hard to define
- So, can't we somehow project onto a lattice, do our things there, then back?
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- Attribution: Dr. Monaghan, SPH meeting 2015, who called it "embedded particle". Then pFEM-2 actually follows this idea
- Results are relevant for any remeshing: particle splitting and merging, field smoothing . . .

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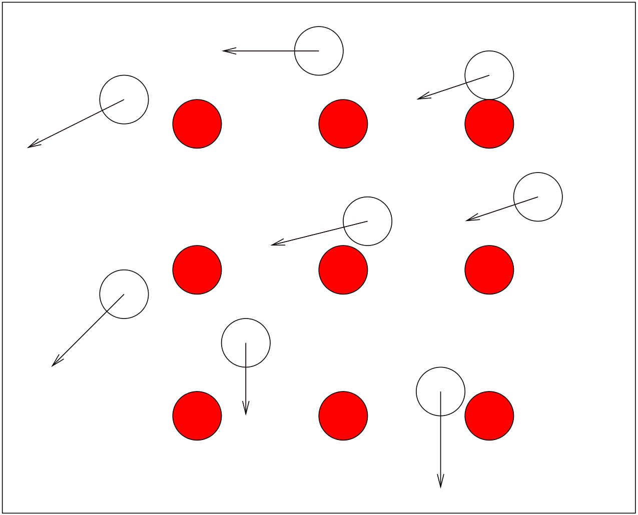
The idea



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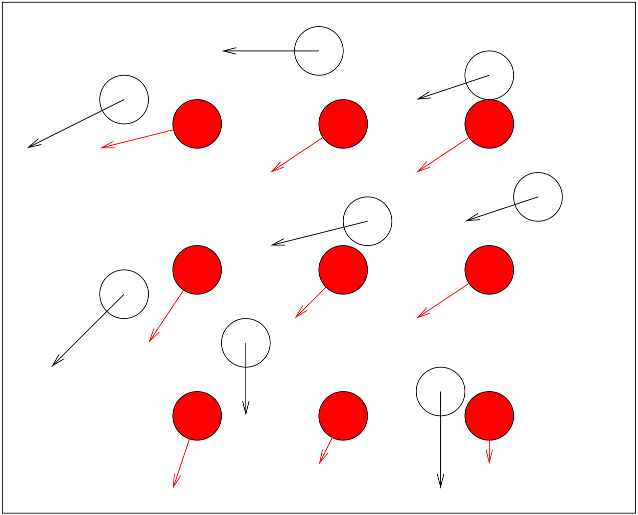

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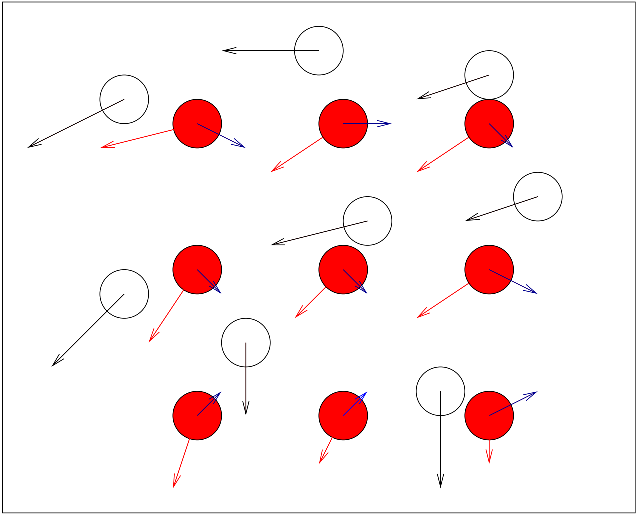

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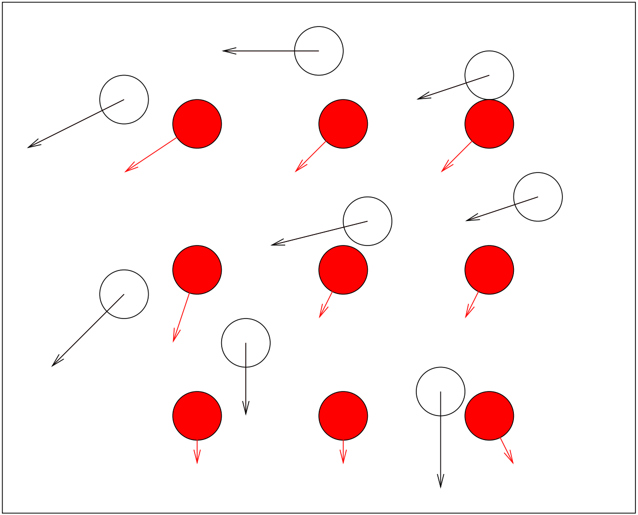

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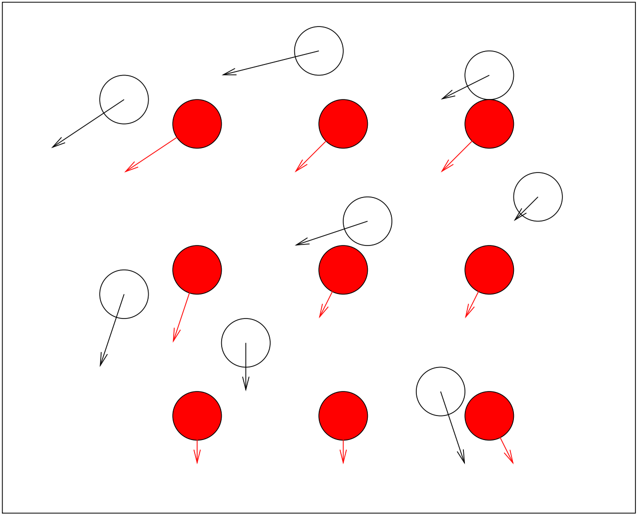

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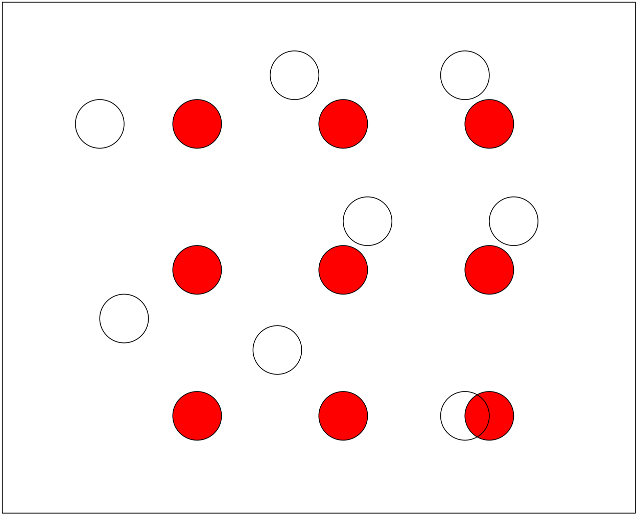

Projecting from the particles

Definition

The particles move about, so we want to interpolate values of fields onto the lattice nodes

This may be achieved with particle basis functions (I know, this usually still requires a mesh)

SPH shape functions may be tried (they must!), but for this talk I'm using p-FEM functions, and an "quad" extension of them

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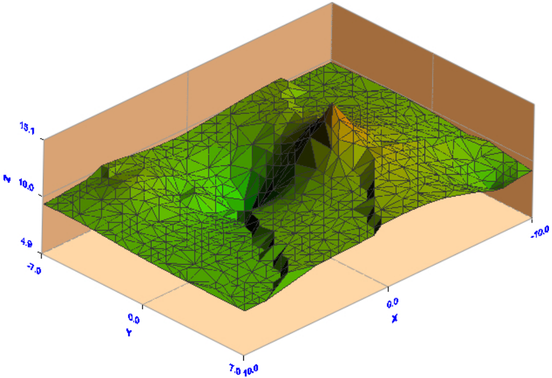
FEM shape functions can be built on particle arrangements at every time step, on the Delaunay triangulation (the dual of the Voronoi diagram)

These will interpolate “linearly” between nodes



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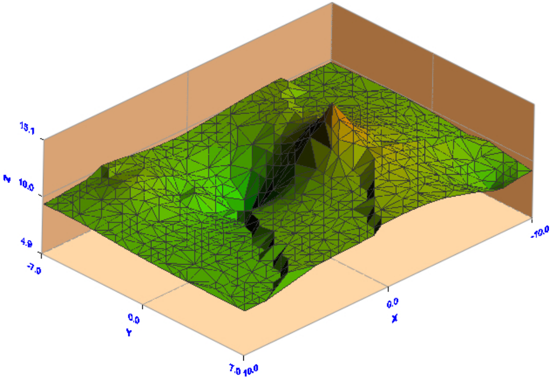

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1D results

In 1D, FEM means just linear interpolation

Let's try our idea, computing the Laplacian of a sine function (periodic b.c.s)

Results are good for the Poisson problem: $h''(x) = f(x)$ given f

Results are not too good for the direct problem: $g(x) = f''(x)$ given f

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1D results, FEM — second derivatives

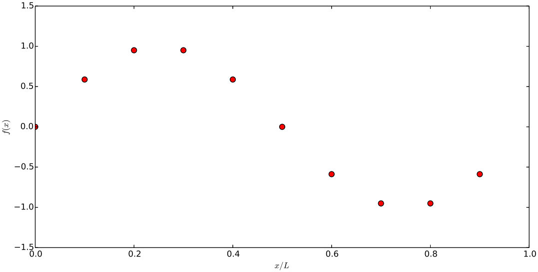

Figure : Original function $f(x)$ on particles

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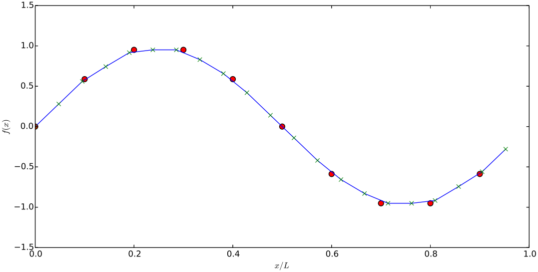

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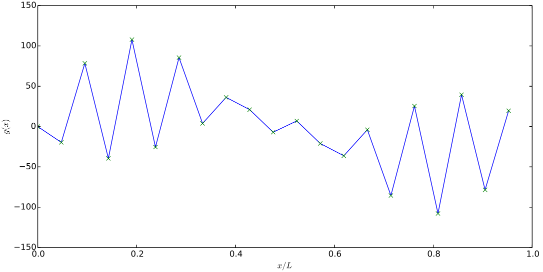

Figure : Second derivative $g(x) = f''(x)$ in lattice

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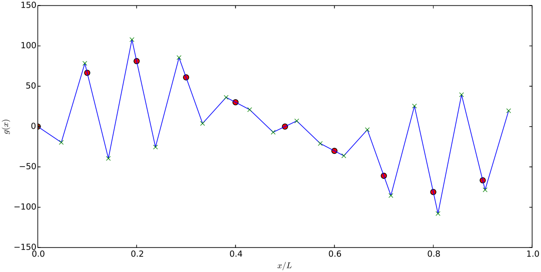

Figure : Second derivative $g(x) = f''(x)$ back on particles

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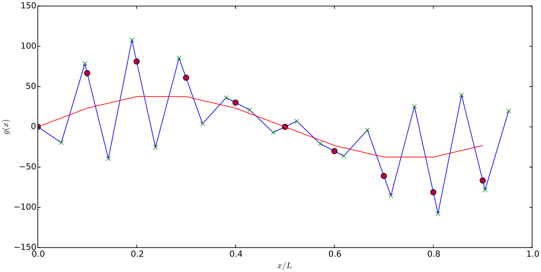

Figure : Second derivative $g(x) = f''(x)$, exact result

Quadratic interpolation

Projection from particles to the lattice seems to be the main culprit

We hereby introduce our (Pep Español, de la Torre, myself) procedure to go from linear to quadratic (sent to journal):

$$\psi_i(\mathbf{r}) = \phi_i(\mathbf{r}) + \sum_{j,k} A_{ijk}\phi_j(\mathbf{r})\phi_k(\mathbf{r})$$

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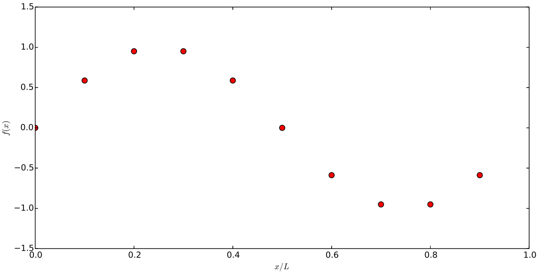

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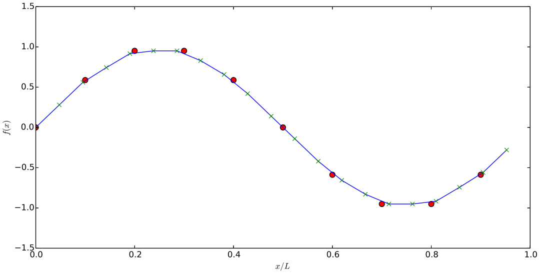

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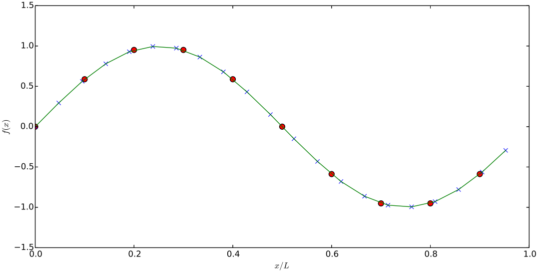

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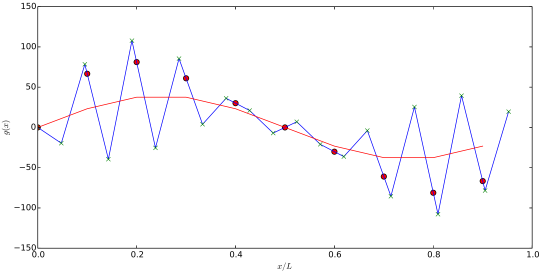

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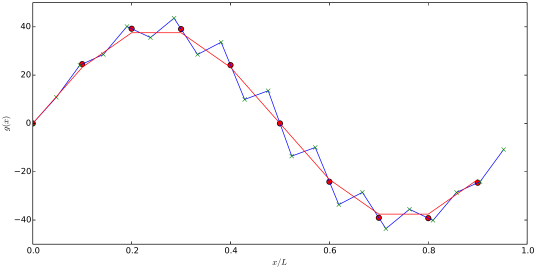

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Going to 2D

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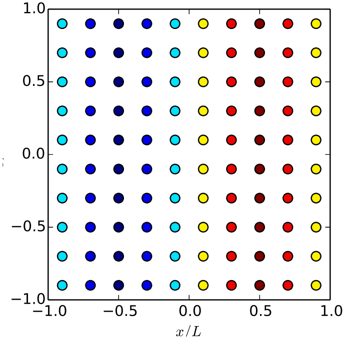

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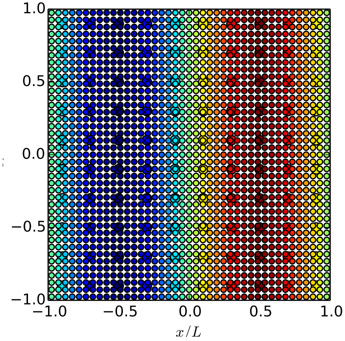

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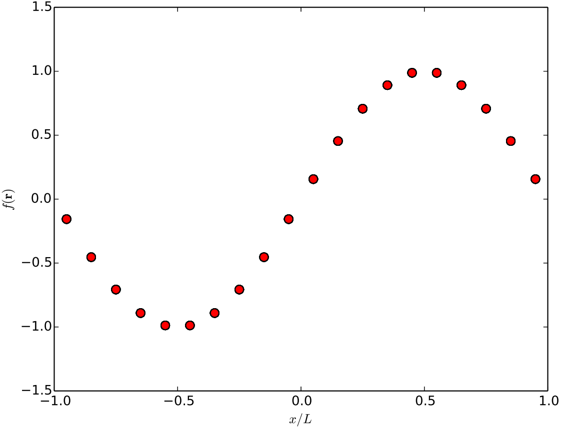

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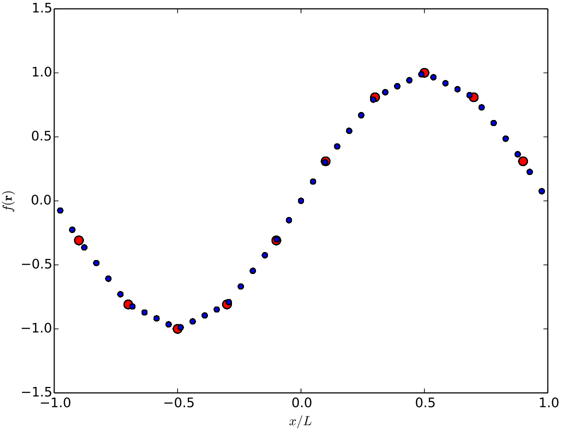

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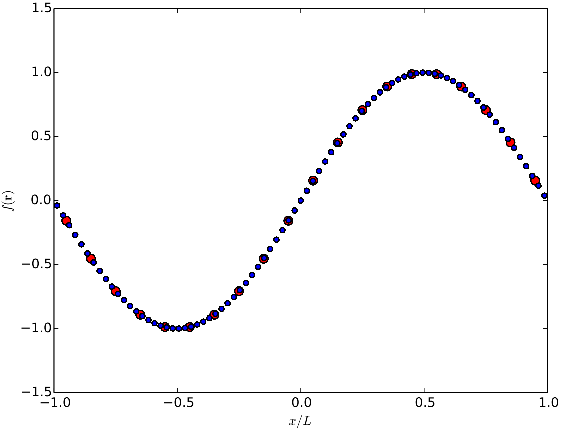

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Taylor-Green vortex sheet

Navier-Stokes for an incompressible fluid:

$$\frac{d\mathbf{u}}{dt} = -\nabla(p/\rho) + \nu\nabla^2\mathbf{u} \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

Taylor-Green solution:

$$u_x = A(t)\sin(\pi x)\cos(\pi y) \tag{3}$$

$$u_y = -A(t)\cos(\pi x)\sin(\pi y) \tag{4}$$

$$A(t) = u_0 \exp(-2\pi^2 \nu t) \tag{5}$$

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Numerical procedure

1. Set up initial conditions
2. Move particles according to $\mathbf{u}_t$
3. Project onto lattice
4. Compute $\mathbf{u}^* = \mathbf{u}_t + (dt)\nu\nabla^2\mathbf{u}_t$
5. Solve PPE $\nabla^2(p/\rho) = 1/(dt)\nabla\cdot\mathbf{u}^*$
6. Compute $\mathbf{u}_{t+1} = \mathbf{u}^* - (dt)\nabla(p/\rho)$
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TG vortices – movie

- lattice vs particles ▸ Link
- quad vs FEM ▸ Link
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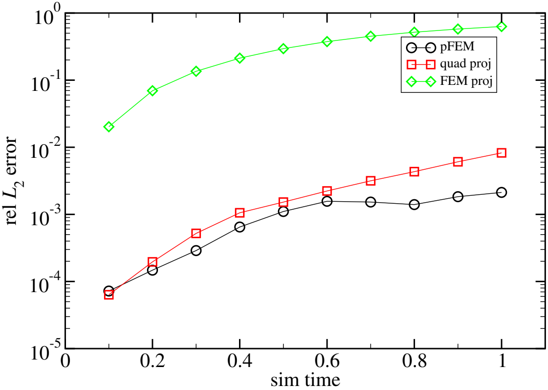
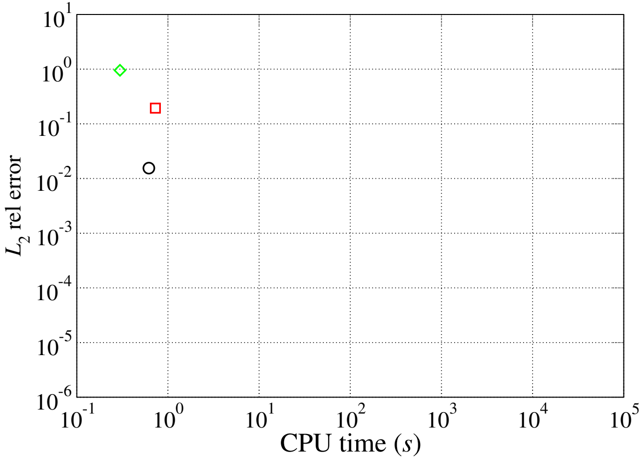
Convergence analysis

Since we know the exact $\mathbf{u} = \bar{\mathbf{u}}$:

$$L_2 := \frac{\sum_i |\bar{\mathbf{u}}_i - \mathbf{u}_i|^2}{\sum_i |\bar{\mathbf{u}}_i|^2}$$



Performance analysis



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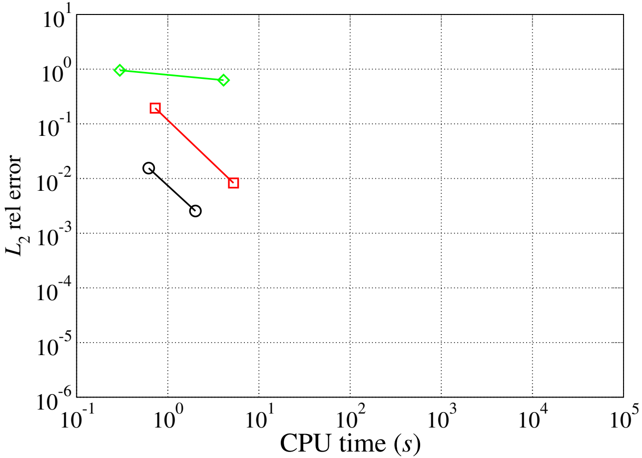

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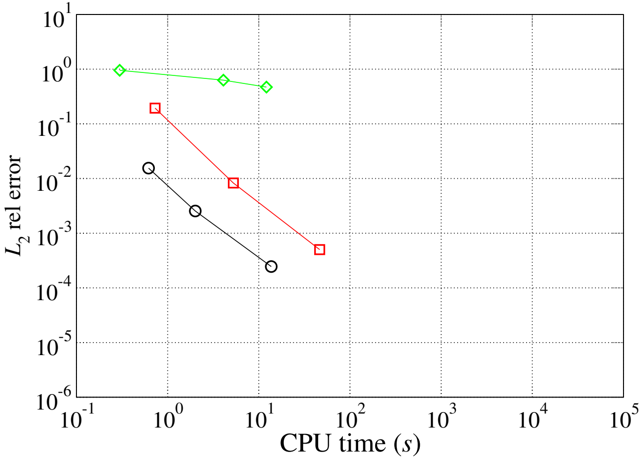

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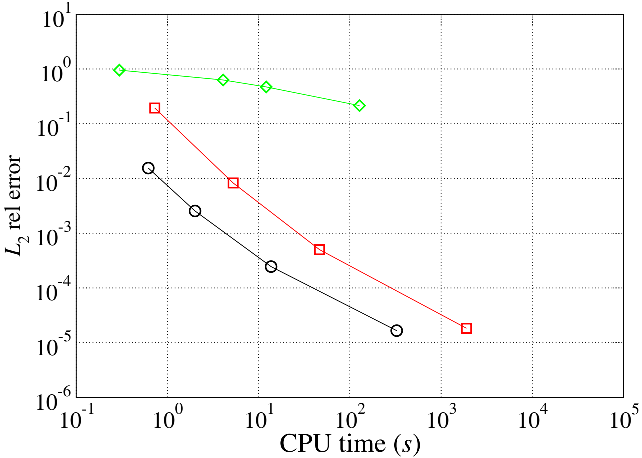

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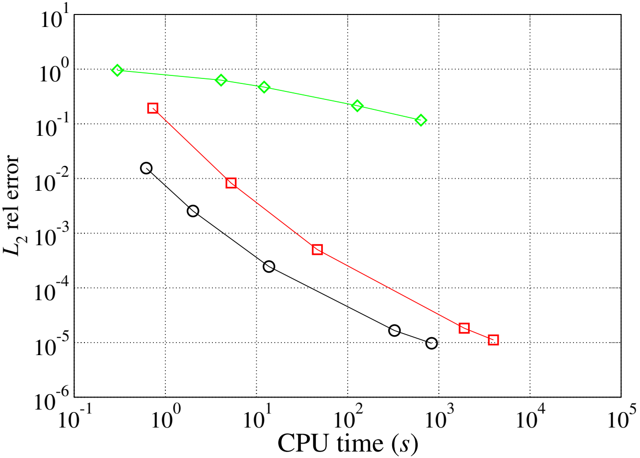

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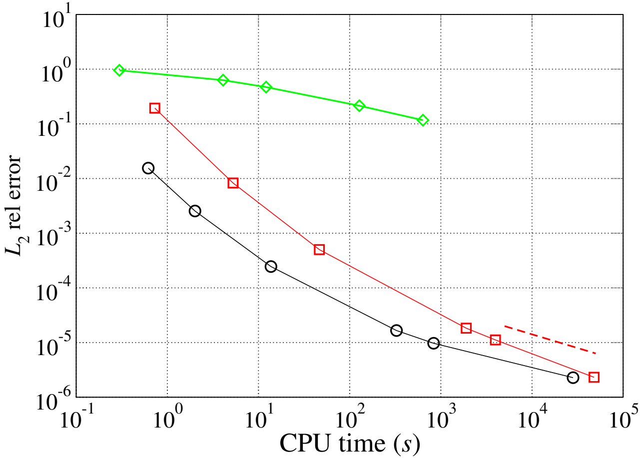

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- As long as the interpolation from particles to lattice is good !
- The application to explicit integration is an open question
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Thanks

For the audience and the organizers

Research has received funding from the Spanish Ministry for Science and Innovation under grant TRA2013-41096-P "Optimización del transporte de gas licuado en buques LNG mediante estudios sobre interacción fluido-estructura."

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37th International Conference on Ocean, Of shore and Arctic Engineering (OMAE 2018)

Madrid, Spain

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VIDEOS

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