Inlet/Outlet BCs along with complex solid geometry treatment

Shahab Khorasanizade, Prof. Joao M.M. Sousa

2nd Iberian SPH, Ourense, Spain

December 3rd 2015

Smoothed Particle Hydrodynamics (SPH)

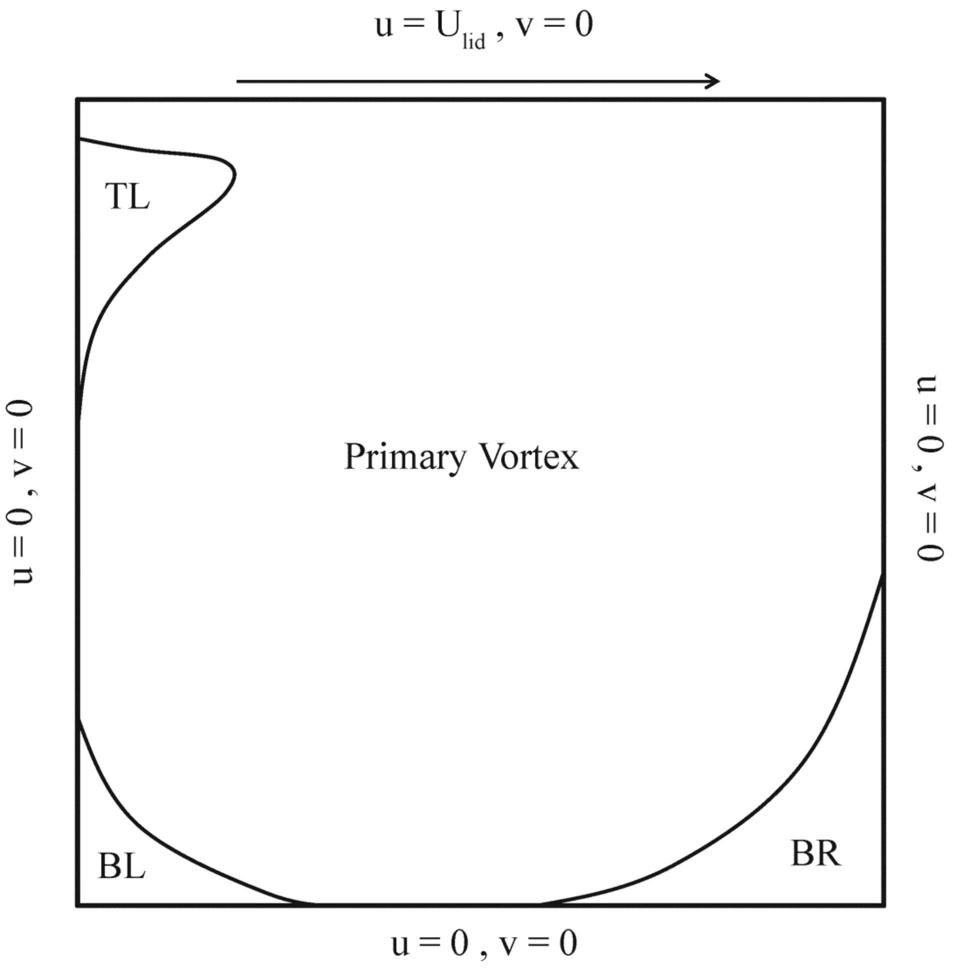
- Governing equations
  - Lagrangian Incompressible Navier-Stokes
  - Prediction-correction algorithm

$$\begin{cases} \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0\,, \\ \dfrac{d\boldsymbol{u}}{dt} = -\dfrac{\boldsymbol{\nabla} p}{\rho} + \nu \boldsymbol{\nabla}^2 \boldsymbol{u} + \boldsymbol{F}\,, \\ \dfrac{d\boldsymbol{r}}{dt} = \boldsymbol{u}\,, \end{cases}$$

$$\nu \boldsymbol{\nabla}^2 \boldsymbol{u}_a = \sum_b \frac{m_b\,(\nu_a + \nu_b)\,\boldsymbol{r_{ab}}.\boldsymbol{\nabla} w_{ab}}{\rho_b r_{ab}^2} \boldsymbol{u}_{ab} \qquad \boldsymbol{\nabla} \cdot \frac{1}{\rho} \boldsymbol{\nabla} p_a = \sum_b \frac{2m_b}{\rho^2} \frac{p_{ab} \boldsymbol{r}_{ab} \cdot \boldsymbol{\nabla} w_{ab}}{r_{ab}^2}$$

$$\boldsymbol{\nabla} p_a = \sum_b \frac{m_b}{\rho_b} (p_b - p_a) \boldsymbol{\nabla} w_{ab} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{u}_a = \sum_b \frac{m_b}{\rho_b} (\boldsymbol{u}_b - \boldsymbol{u}_a) \cdot \boldsymbol{\nabla} w_{ab}$$

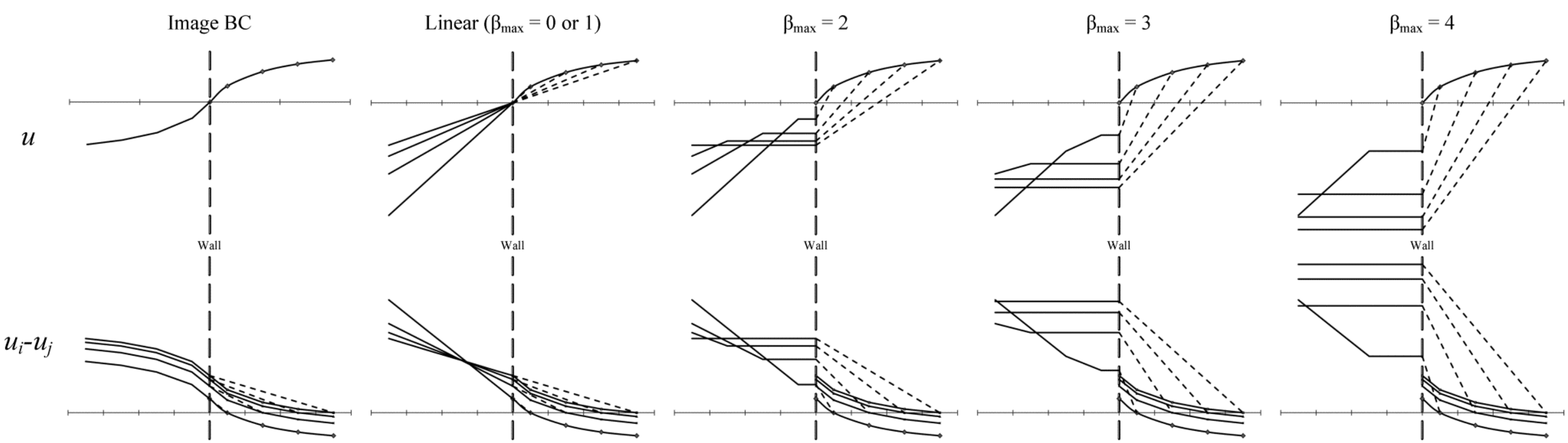
Solid BC Developments



Solid BC Developments

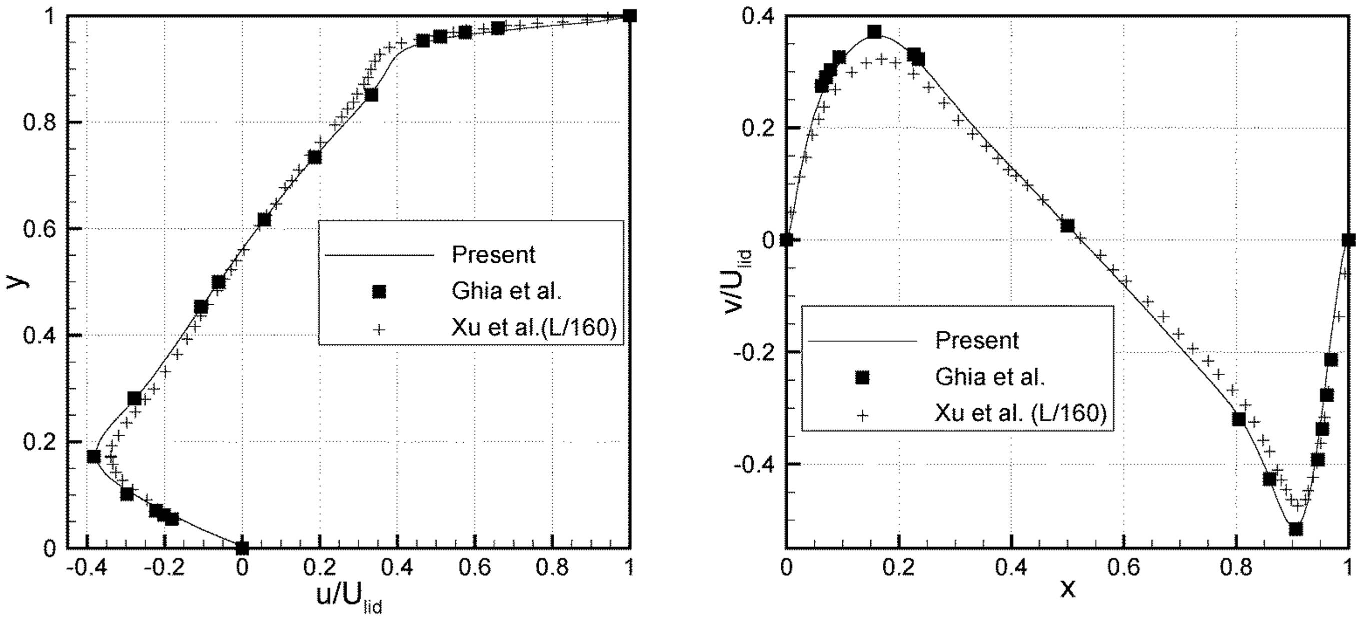
- Solid walls, no-slip condition
  - Image particle
    - Velocity treatment

$$\beta = max\left(\beta_{max}, 1 + \frac{d_{BC}}{d_f}\right) \qquad \boldsymbol{u}_f - \boldsymbol{u}_{BC} = \beta(\boldsymbol{u}_f - \boldsymbol{u}_{wall})$$



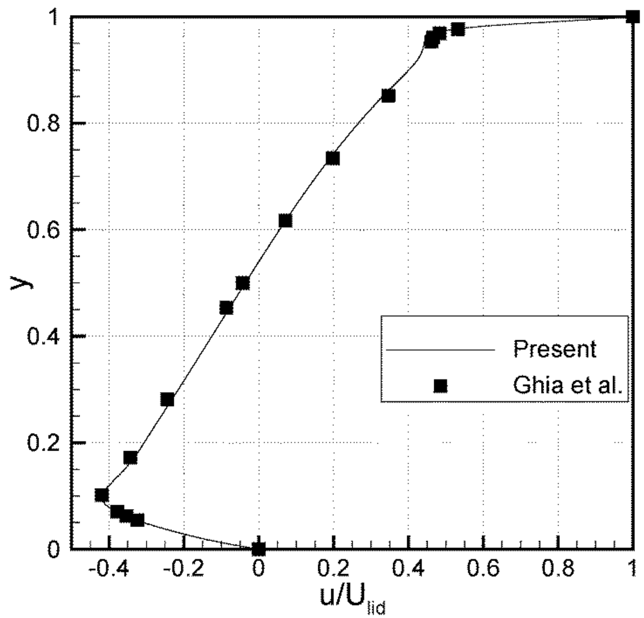
Solid BC Results

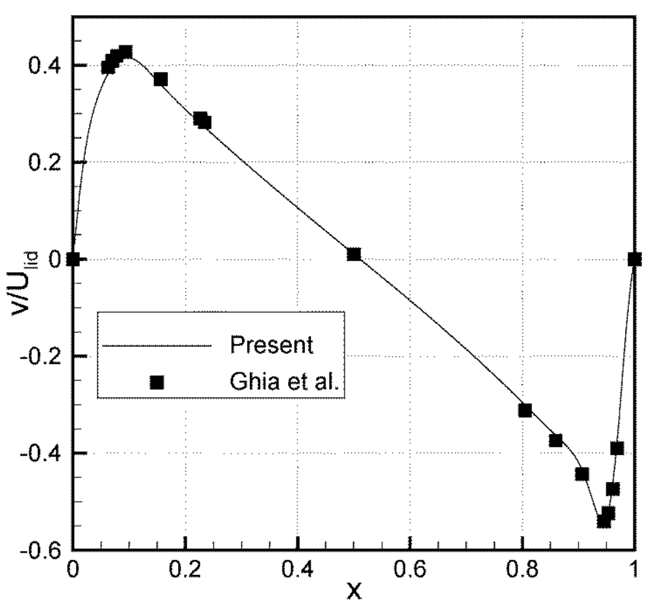
- Re = 1000, L/130



Solid BC Results

- Re = 3200, L/200





Open BC Developments

- Periodic
  - Constant Driving Force
    - Plane channel

$$F \propto M \quad (1)$$

  - Relaxed Driving Force (RDF)
    - F is driving force
    - M is flow rate
    - γ is relaxation coefficient

$$F^{k+1} = F^k \left[\left(\frac{M_0}{M_k} - 1 \right) \gamma + 1 \right] \quad (2)$$

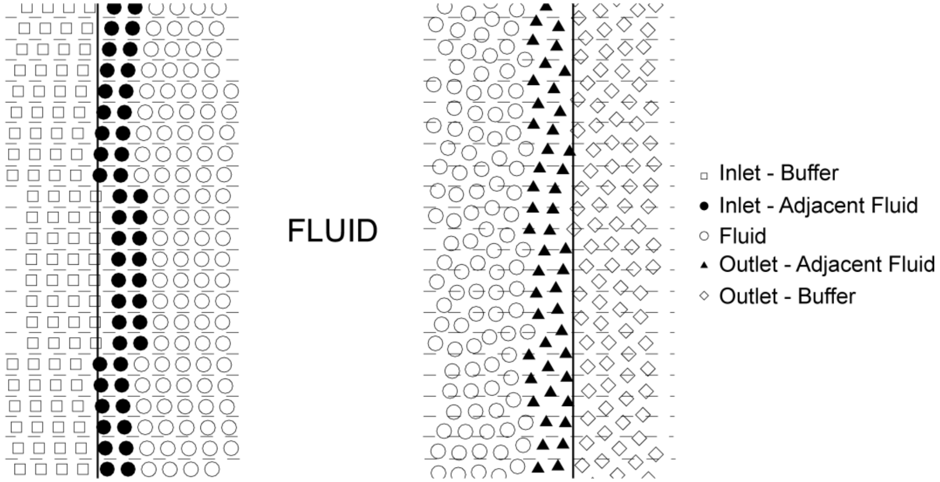
Open BC Developments

- Inlet/outlet
  - Periodic BC simulations
  - Buffer creation
    - Compact support size
    - Outlet particle penetration
    - P from closest layer

- $\boldsymbol{u}_{outlet}$ Orlanski (1976) traveling wave (FV)

$$\frac{\partial \varphi}{\partial t} + c\frac{\partial \varphi}{\partial x_1} = 0$$

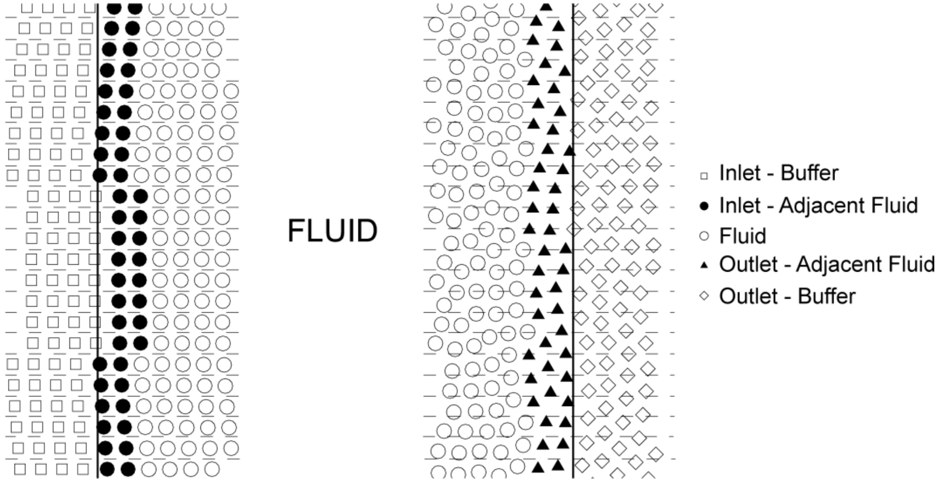
$$\varphi_B^{n+1} = \varphi_B^n - \frac{U_0 \Delta t}{\Delta x^1}\left(\varphi_B^n - \varphi_F^n\right)$$



Open BC Developments

- Inlet/outlet
  - Periodic BC simulations
  - Buffer creation
    - Compact support size
    - Outlet particle penetration
    - P from closest layer

- frozen $\boldsymbol{u}$ once fluid → outlet

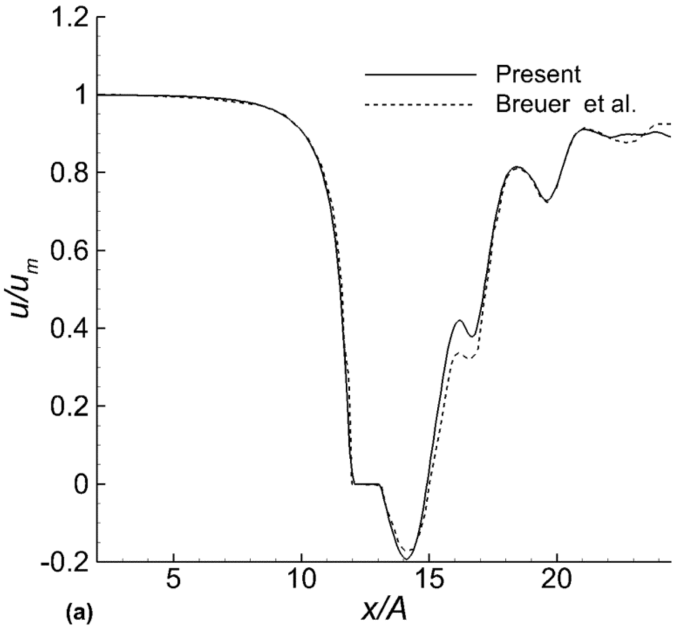


$$\left[-p\boldsymbol{n} + (\rho\nu)\boldsymbol{n}\cdot\left(\boldsymbol{\nabla u} + \boldsymbol{\nabla u}^{\mathrm{T}}\right)\right]\Big|_{\Gamma} = \boldsymbol{c}$$

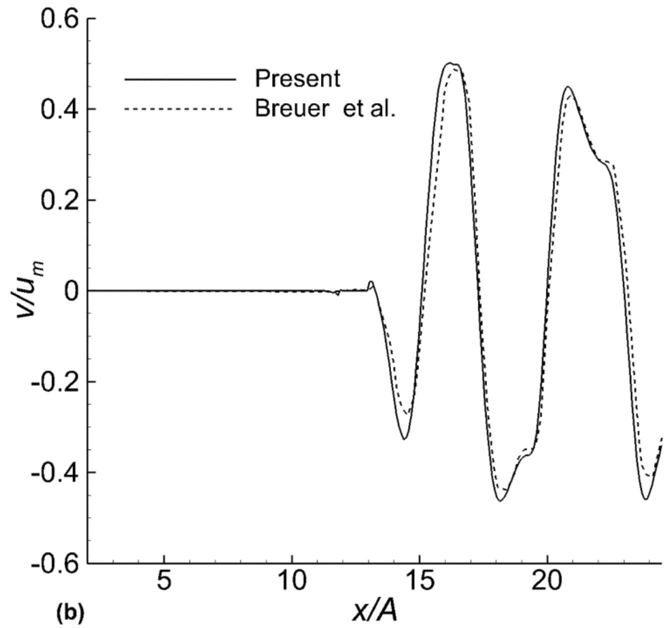
$$\begin{cases} \boldsymbol{F} = -\dfrac{1}{\rho}\boldsymbol{\nabla} p_F \\ p_{total} = p_{solver} + p_F \end{cases}$$

Inflow/Outflow Results

- Flow over square obstacle in channel
  - $\beta = 1/8$, Re = 100
  - Modified Shifting algorithm



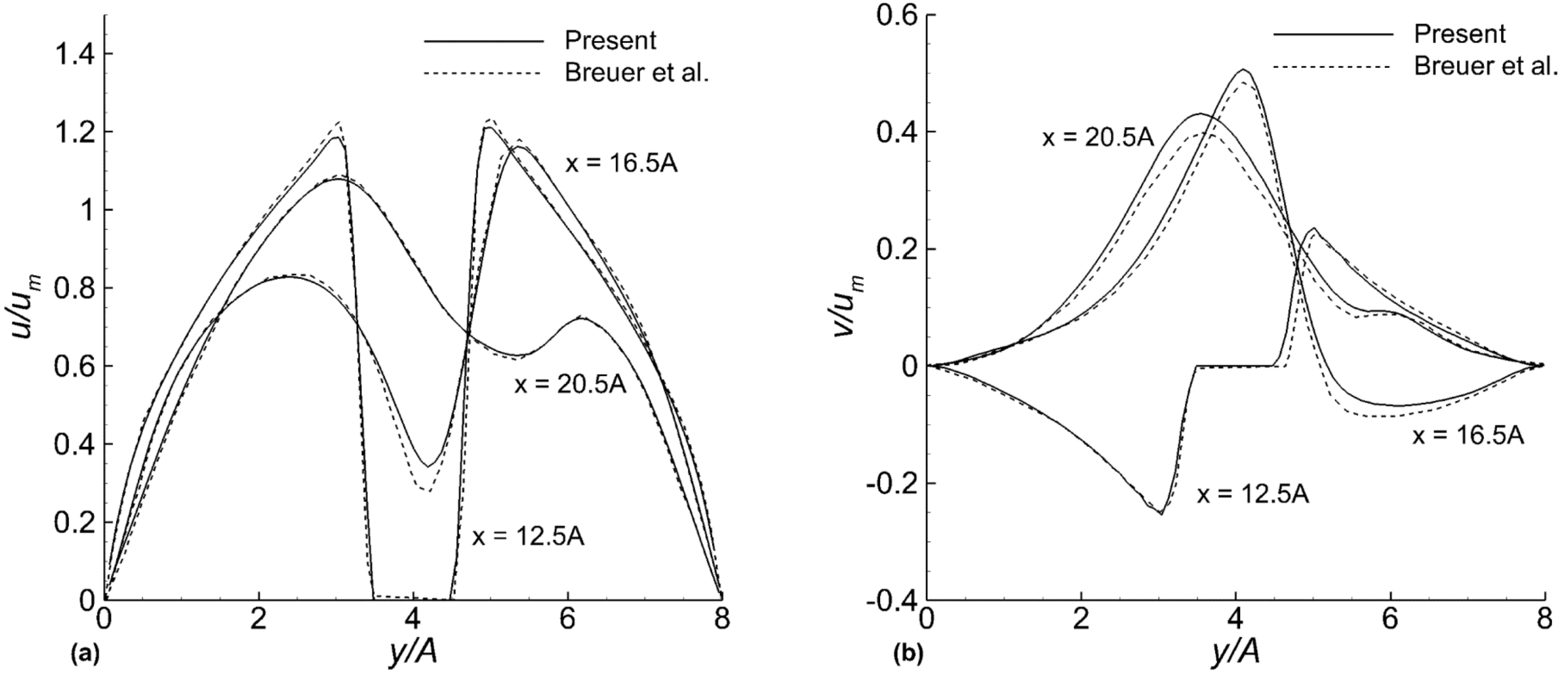
(a)



(b)

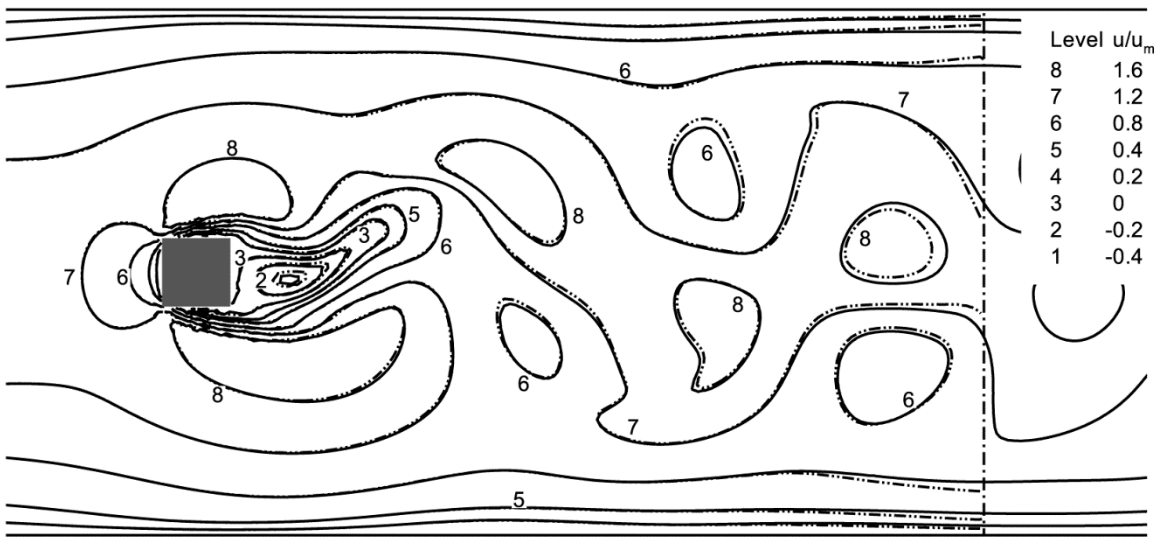
Inflow/Outflow Results

- Flow over square obstacle in channel
  - $\beta = 1/8$, Re = 100
  - Modified Shifting algorithm



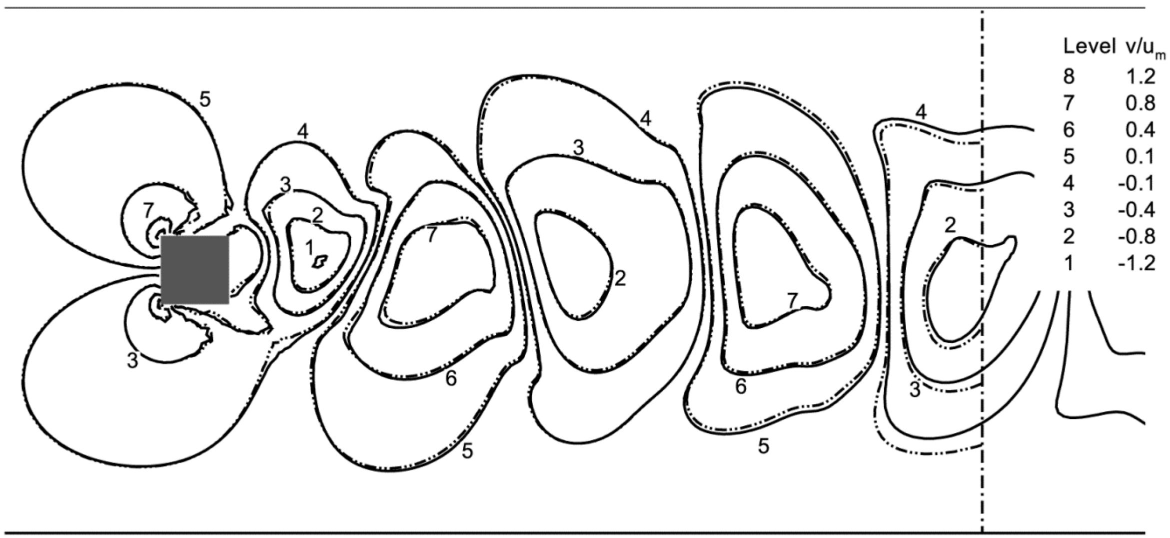
Inflow/Outflow Results

- Flow over square obstacle in channel
  - β = 1/8, Re = 200
  - Modified Shifting algorithm



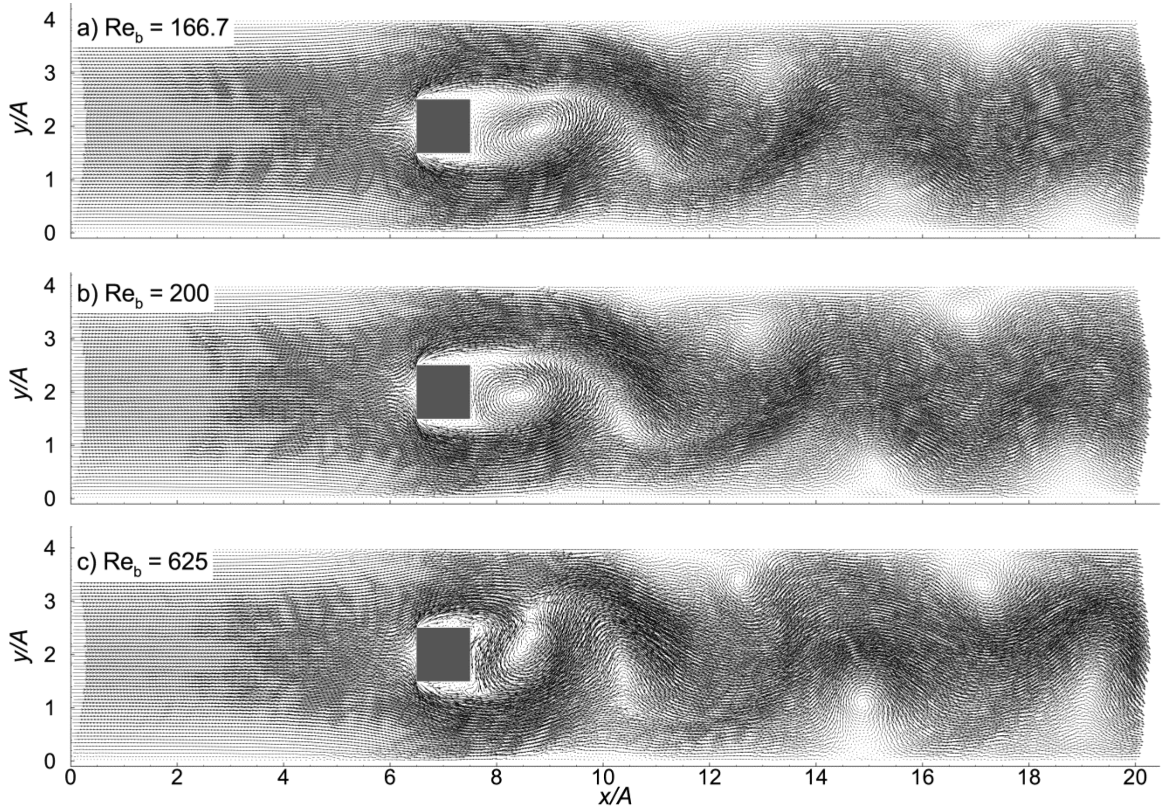
Inflow/Outflow Results

- Flow over square obstacle in channel
  - $\beta = 1/8$, Re = 200
  - Modified Shifting algorithm



Inflow/Outflow Results

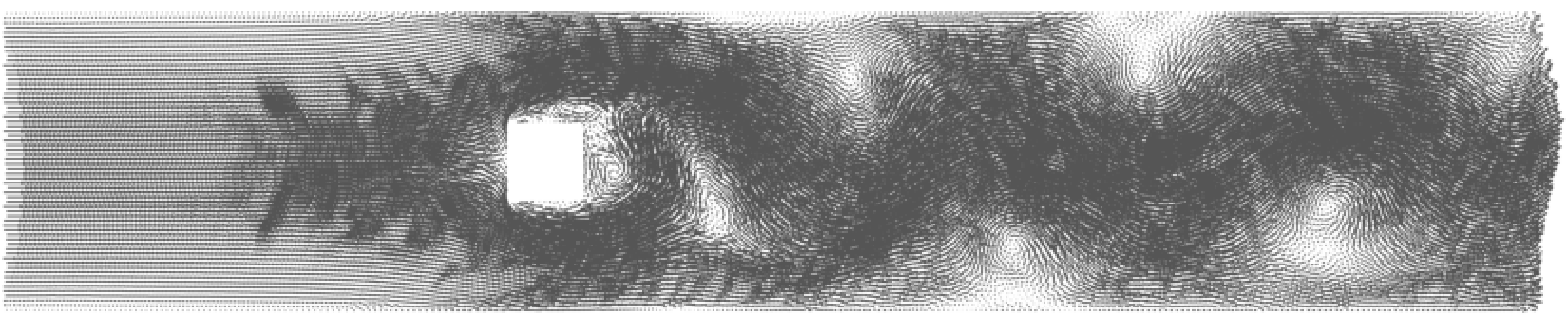
- Flow over square obstacle in channel
  - $\beta = 1/4$
  - Modified Shifting algorithm



Inflow/Outflow Results

- Flow over square obstacle in channel
  - $\beta = 1/4$
  - Modified Shifting algorithm

  $Re_b = 625$



Instabilities

- Particle shifting

$$\begin{cases} \delta \boldsymbol{r}_{ii'} = C'\boldsymbol{R}_i \ , \\ \boldsymbol{R}_i = \sum_{j=1}^{N_i} \frac{\boldsymbol{r}_{ij}}{r_{ij}^3} \bar{r}_i^2 \ , \\ \bar{r}_i = \frac{1}{N_i} \sum_{j=1}^{N_i} r_{ij} \ , \end{cases} \qquad C' = u_{max} \, \Delta t C$$

$$\begin{cases} a = u_{max} \Delta t C \\ b = u_{max,i}^{local} \Delta t C \\ d = \bar{r}_i C_{FVPM} \\ f = \max(b, d) \\ C' = \min(a, f) \end{cases}$$

$$f_{i'} = f_i + \delta \boldsymbol{r}_{ii'} . (\boldsymbol{\nabla} f)_i$$

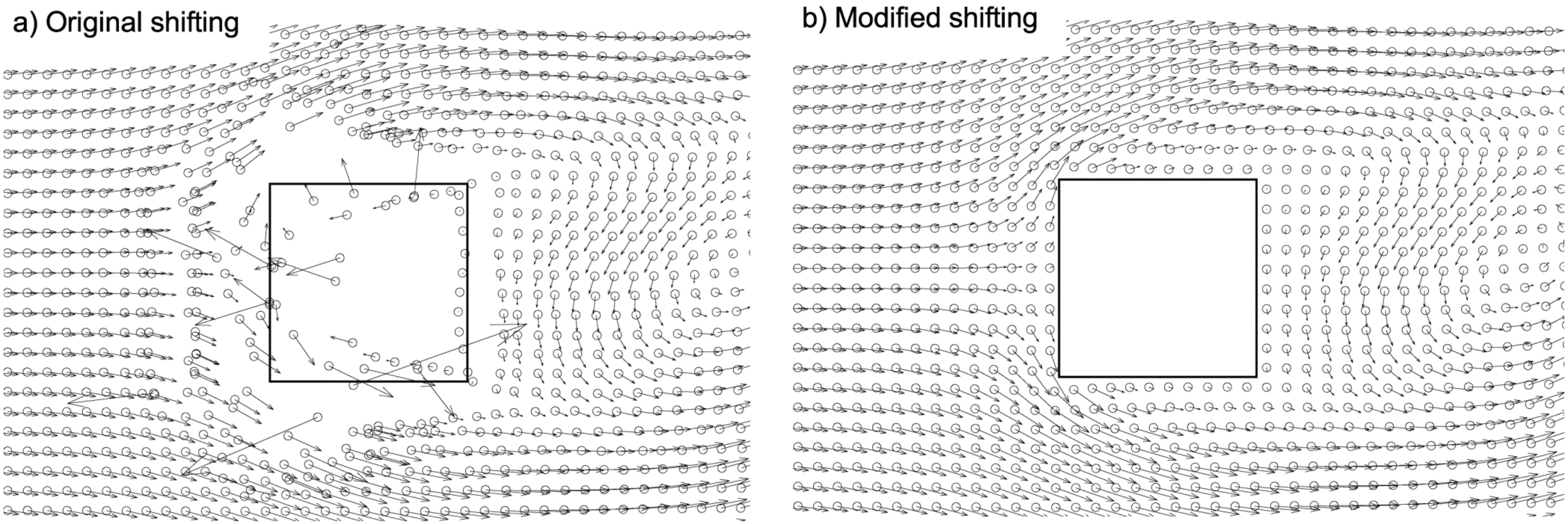
$$C = 0.04$$

$$C_{FVPM} = 0.0001$$

$$\boldsymbol{r}_{i'} = \boldsymbol{r}_i^{n+1} + \delta \boldsymbol{r}_{ii'}$$

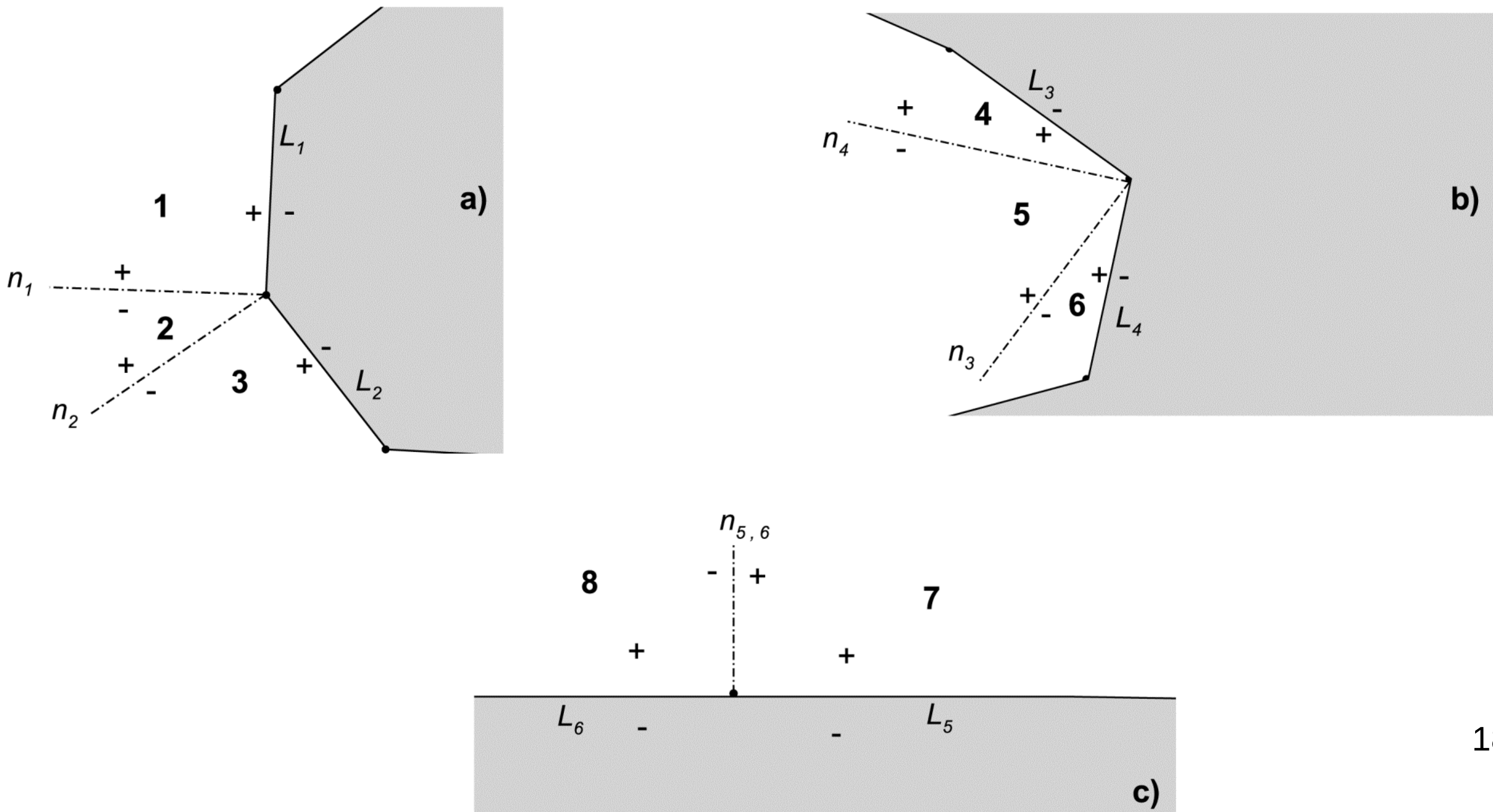
Instabilities

- Particle shifting



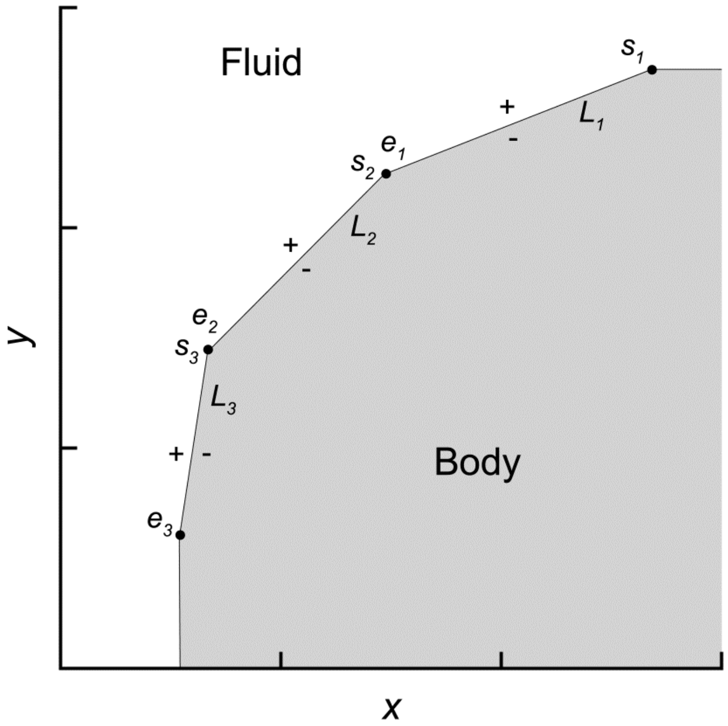
Solid BC Developments

- Complex solid boundaries
  - o Tessellation



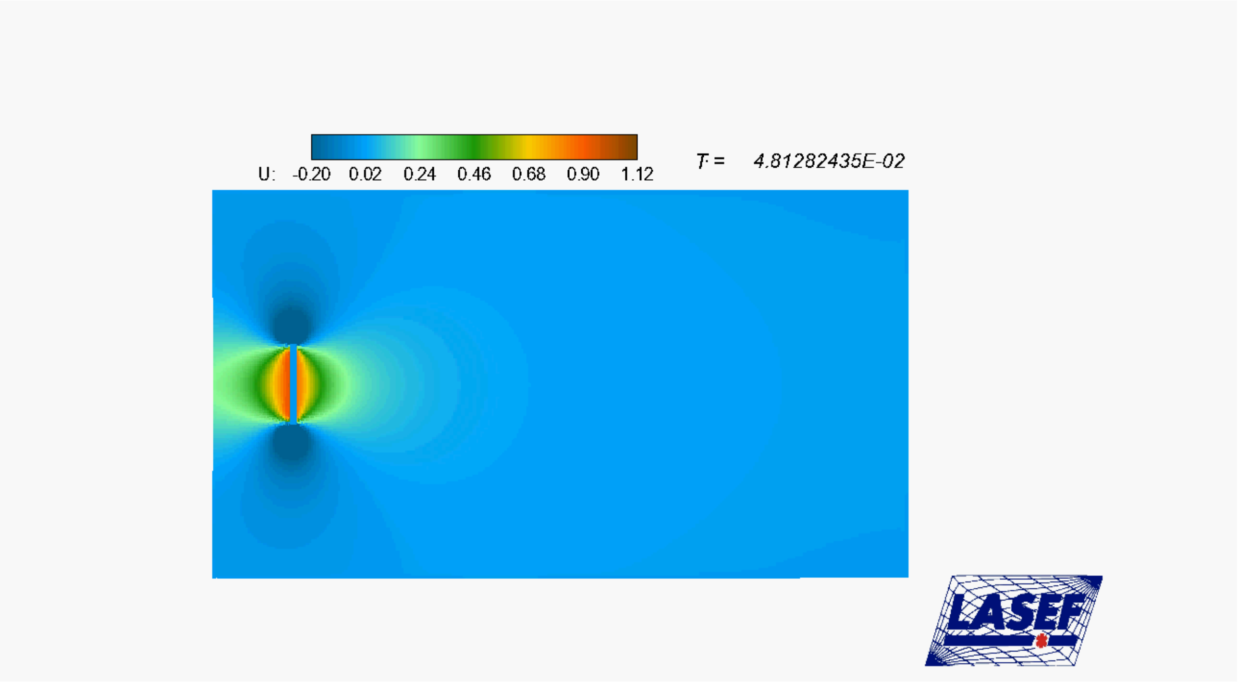
Solid BC Developments

- Complex solid boundaries
  - Tessellation



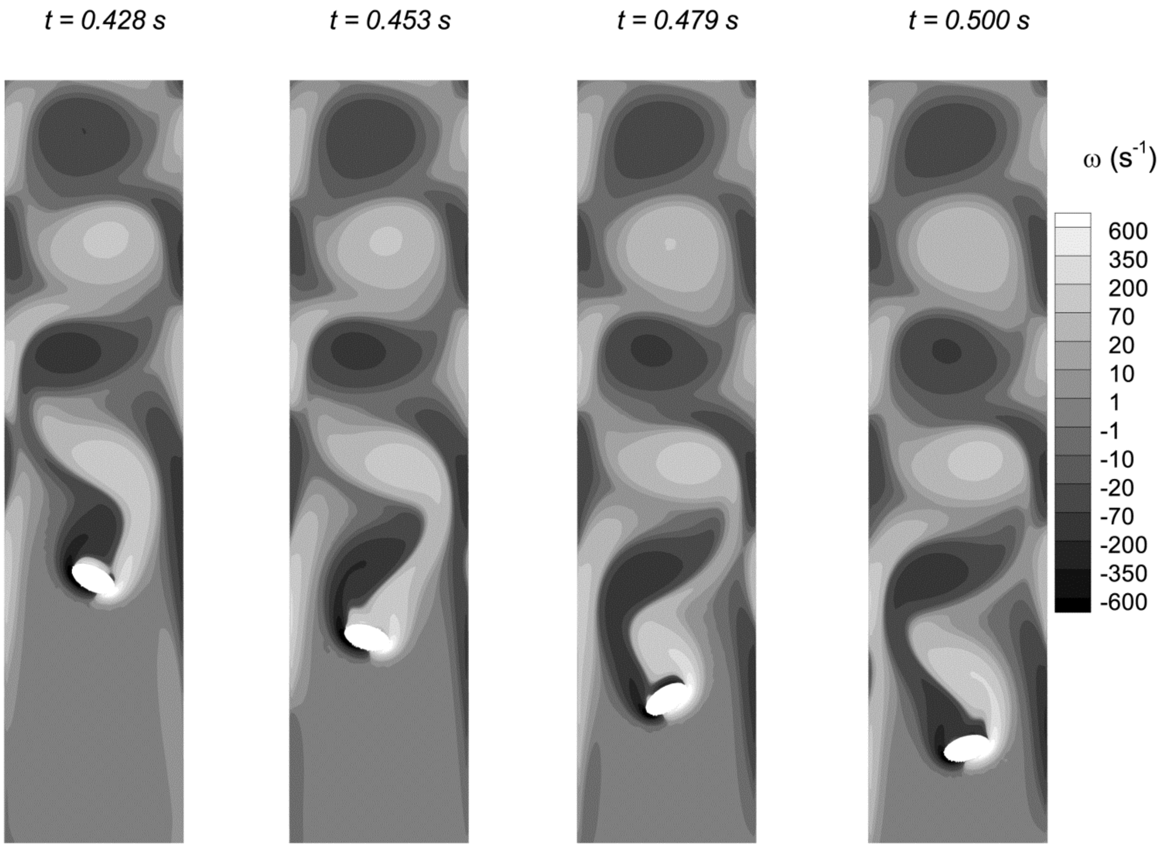
Complex solid boundaries

- Impulsive motion of a thin plate
  - *Re* = 20



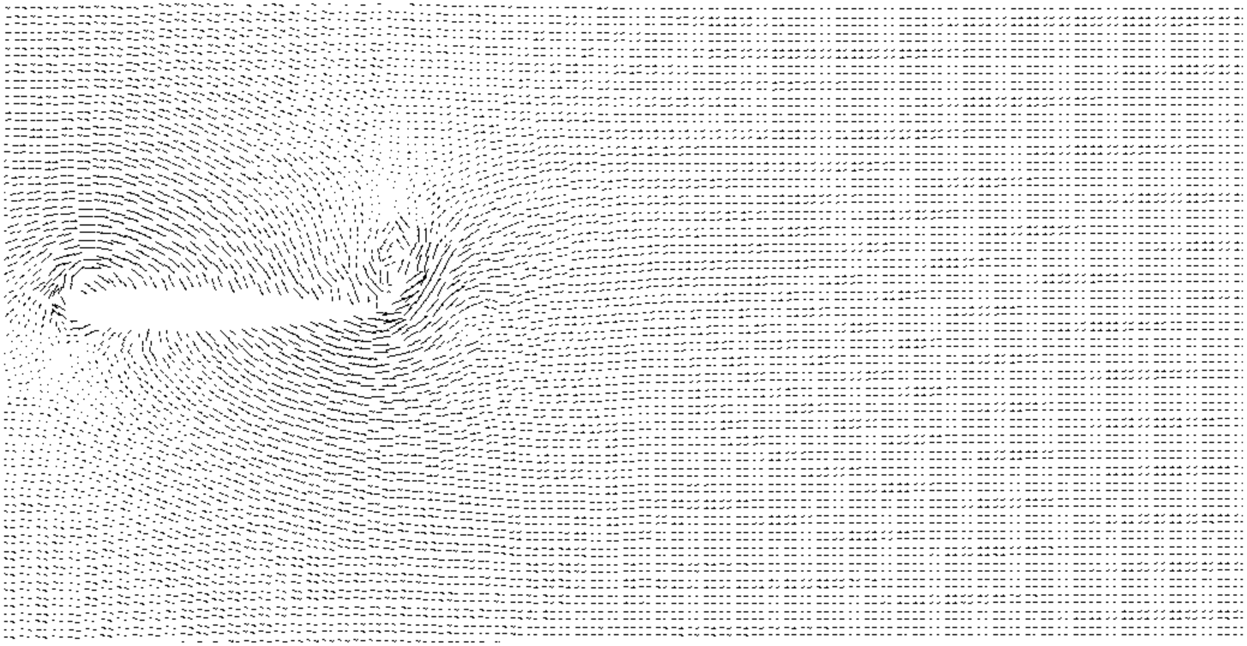
Complex solid boundaries

- Sedimentation of an ellipse
  - $\gamma = 2.0$



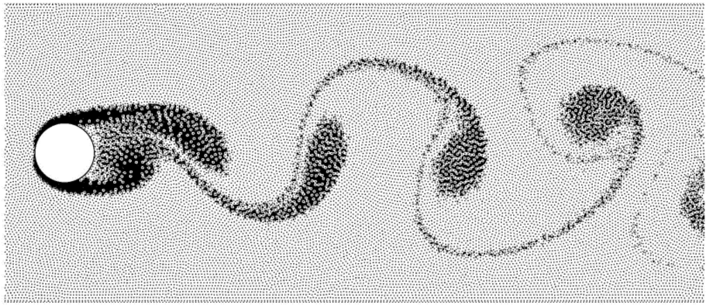
Complex solid boundaries

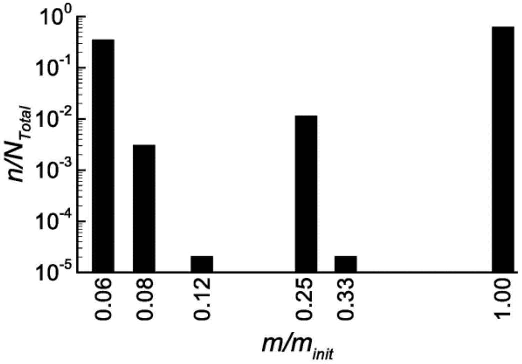
- Plunging Airfoil (NACA0012)
  - $k = 12.3$, $h = 0.12$

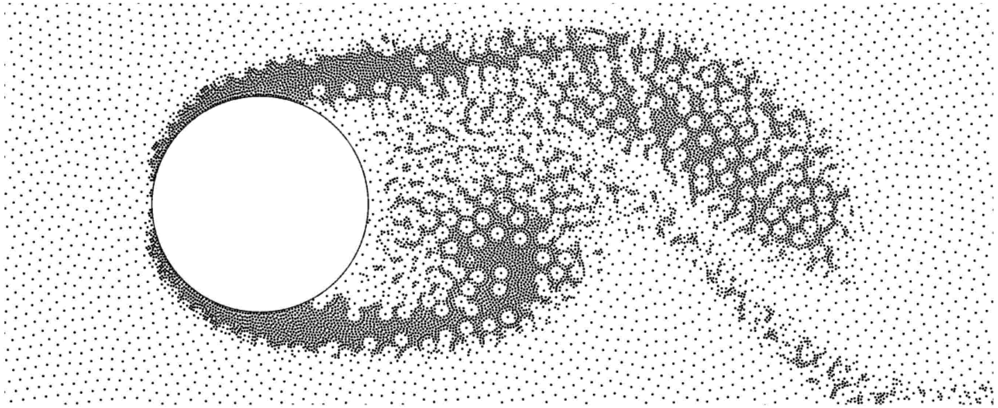


Complex solid boundaries & Adaptive

- Circular object
  - o Re = 100

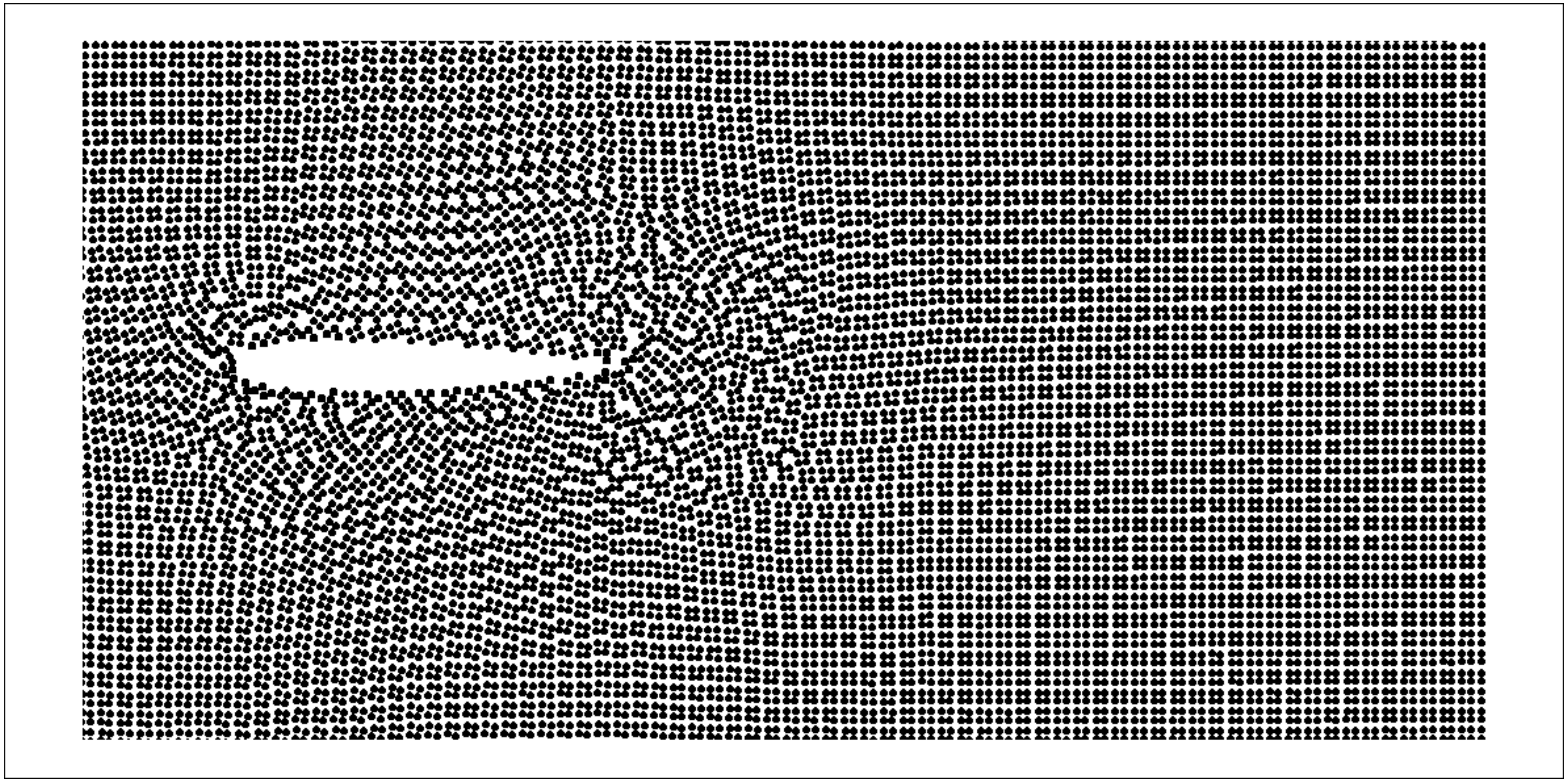






Complex solid boundaries

- Plunging Airfoil (NACA0012)
  - $k = 12.3$, $h = 0.12$



References

- Khorasanizade, Sh. and Sousa, J.M.M. (2014), A detailed study of lid-driven cavity flow at moderate Reynolds numbers using Incompressible SPH, Int. J. Numer. Meth. Fluids, 76; pages 653–668, doi: 10.1002/fld.3949

- Khorasanizade, Sh. and Sousa, J.M.M. (2015a), An innovative open boundary treatment for incompressible SPH, Int. J. Numer. Meth. Fluids, doi: 10.1002/fld.4074.

- Khorasanizade, Sh. and Sousa, J.M.M. (2015b), A two-dimensional Segmented Boundary Algorithm for complex moving solid boundaries in Smoothed Particle Hydrodynamics, Comput. Phys. Comm., doi: 10.1016/j.cpc.2015.10.025.

- Khorasanizade, Sh. and Sousa, J.M.M. (2015c), Dynamic flow-based particle splitting in smoothed particle hydrodynamics, Int. J. Num. Meth. Engng., doi: 10.1002/nme.5128